

Radiative-corrections for the MUSE experiment

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for the MUSE Collaboration

Supported in parts by the U.S. National Science Foundation: NSF PHY-1812382. The MUSE experiment is supported by the U.S. Department of Energy, the U.S. National Science Foundation, the Paul Scherrer Institute, and the US-Israel Binational Science Foundation.

Radiative Corrections Workshop, Stoney Brook (online), July 9–10, 2020

MUon Scattering Experiment (MUSE) at PSI



Direct test of μp and $e p$ interactions in a scattering experiment:

- higher precision than previously for μp ,
- low- Q^2 region for sensitivity to the **proton charge radius**,
 $Q^2 = 0.002$ to 0.07 GeV^2 ,
- with μ^+, μ^- and e^+, e^- to study possible **2γ mechanisms**,
- with μp and $e p$ to have direct **μ/e comparison**.

MUSE

$$e^- p \rightarrow e^- p$$

$$e^+ p \rightarrow e^+ p$$

$$\mu^- p \rightarrow \mu^- p$$

$$\mu^+ p \rightarrow \mu^+ p$$

Projected MUSE proton charge-radius results

How different are the e/ μ radii?

(truncation error largely cancels)

Sensitivity to differences in
extracted e/ μ radii:

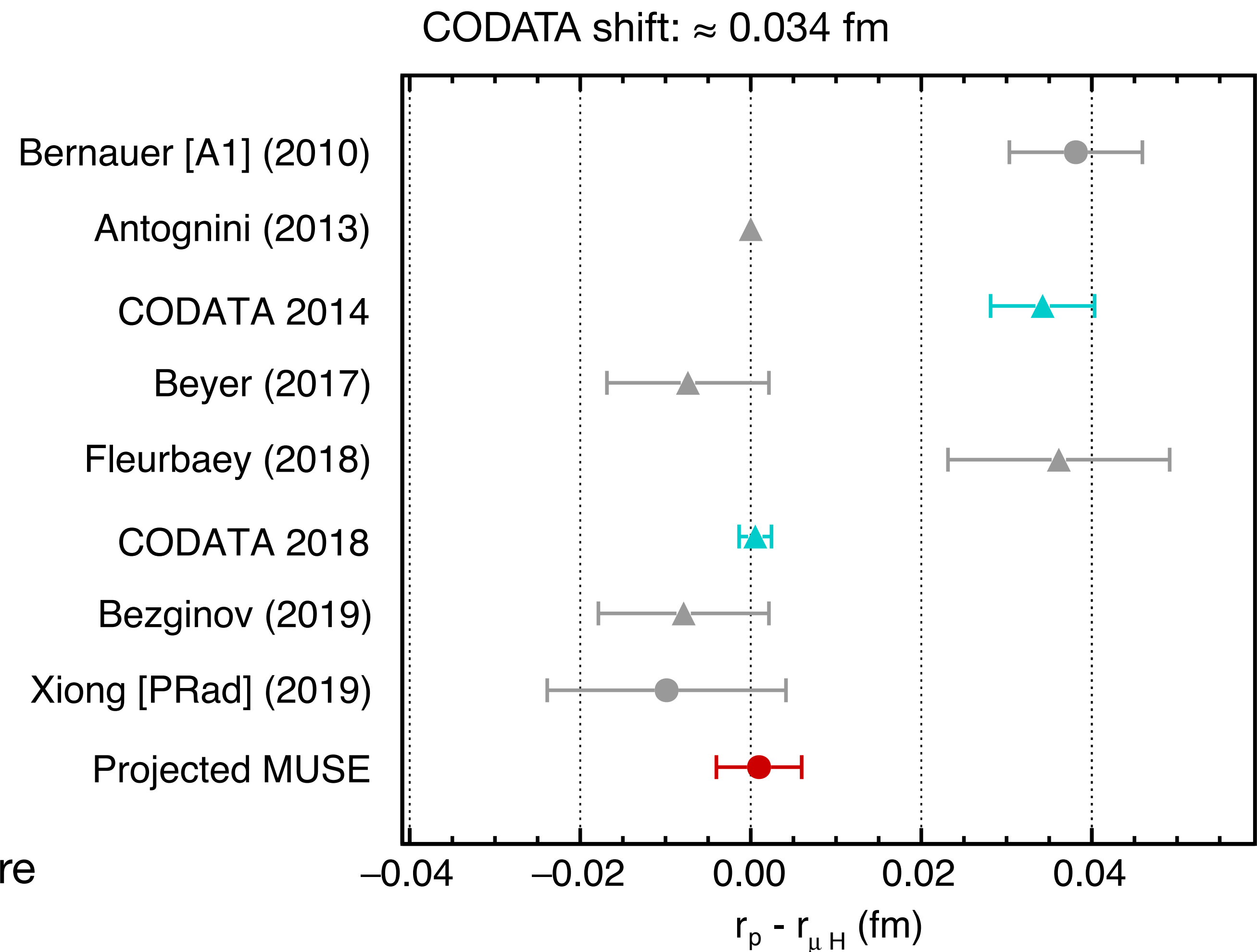
$$\sigma(r_e - r_\mu) \approx 0.005 \text{ fm}$$

What is the radius?

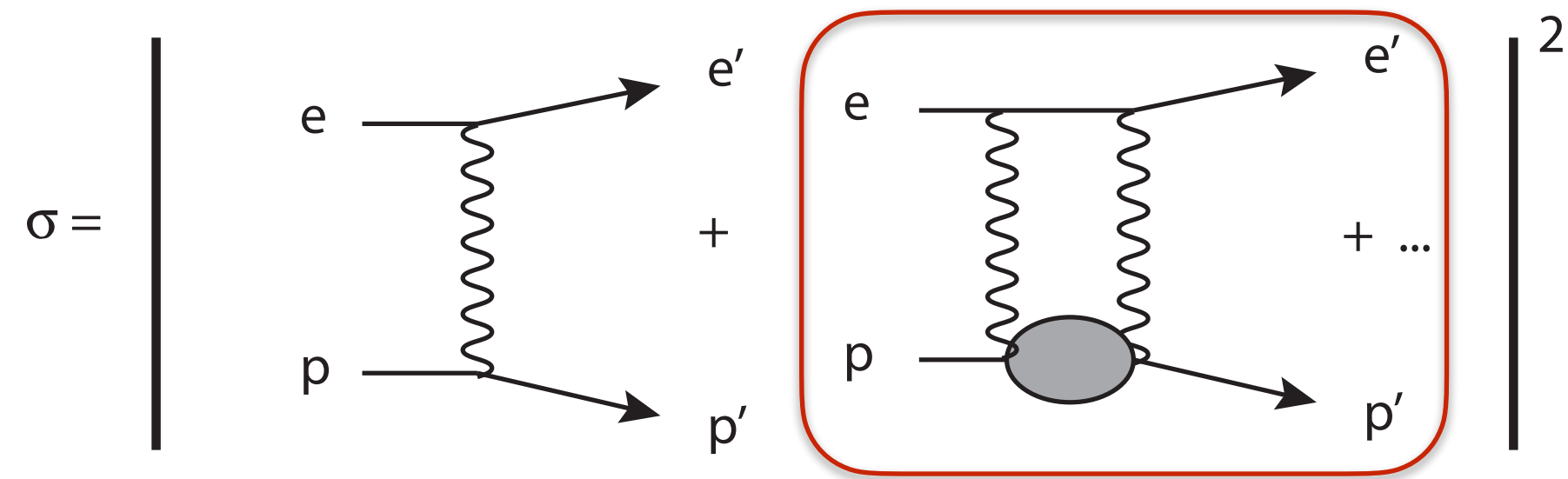
Absolute values of extracted
e/ μ radii (assuming no +/-
difference seen):

$$\sigma(r_e), \sigma(r_\mu) \approx 0.008 \text{ fm}$$

Comparisons of, e.g., **e to μ** or of **μ^+ to μ^-** are
insensitive to many of the systematics



MUSE provides a high precision test of two-photon exchange for electrons and muons at low Q^2



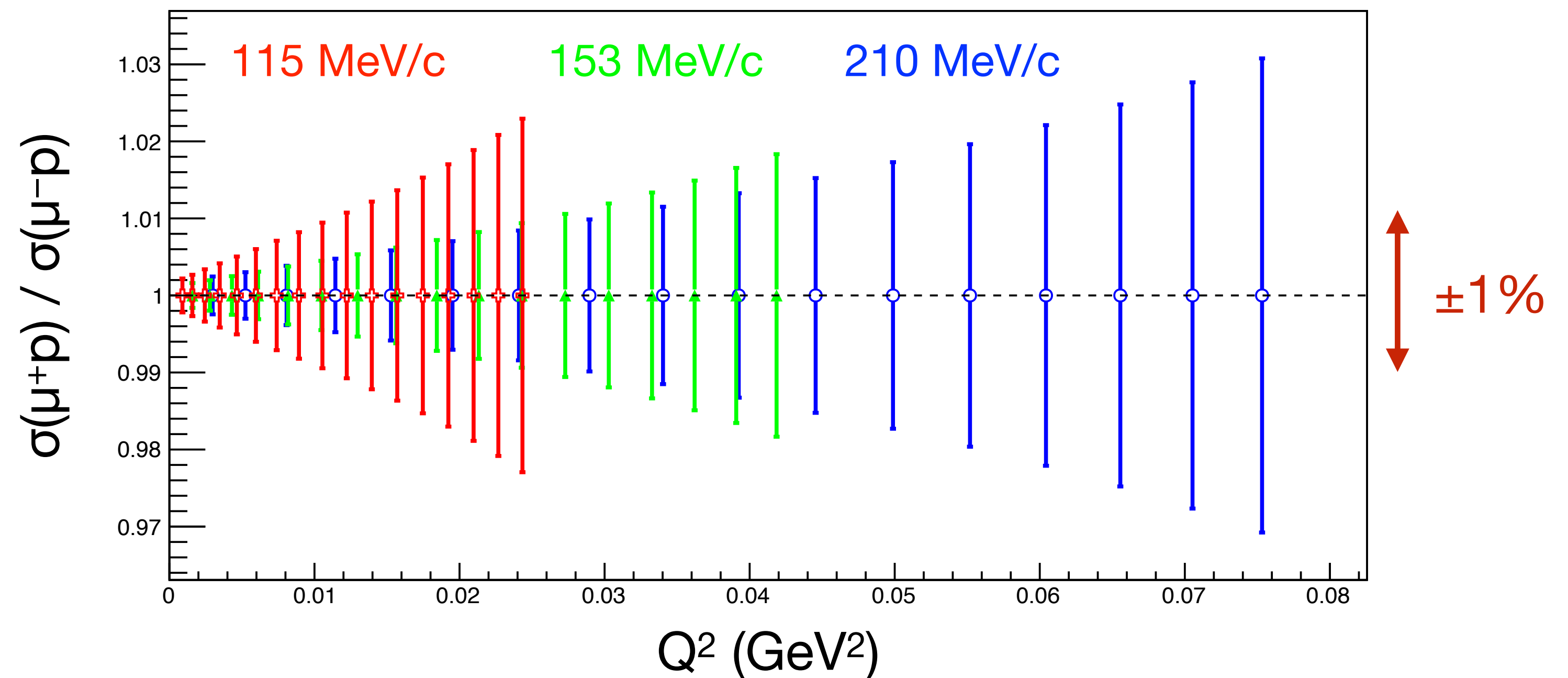
TPE: largest theoretical uncertainty in low-energy proton structure.

Projected relative uncertainty in the ratio of μ^+p to μ^-p elastic cross sections. Estimated systematics: 0.2%.

$$\sigma_{e^\pm p} = |\mathcal{M}_{1\gamma}|^2 \pm 2\Re\{\mathcal{M}_{1\gamma}^\dagger \mathcal{M}_{2\gamma}\} + \dots$$

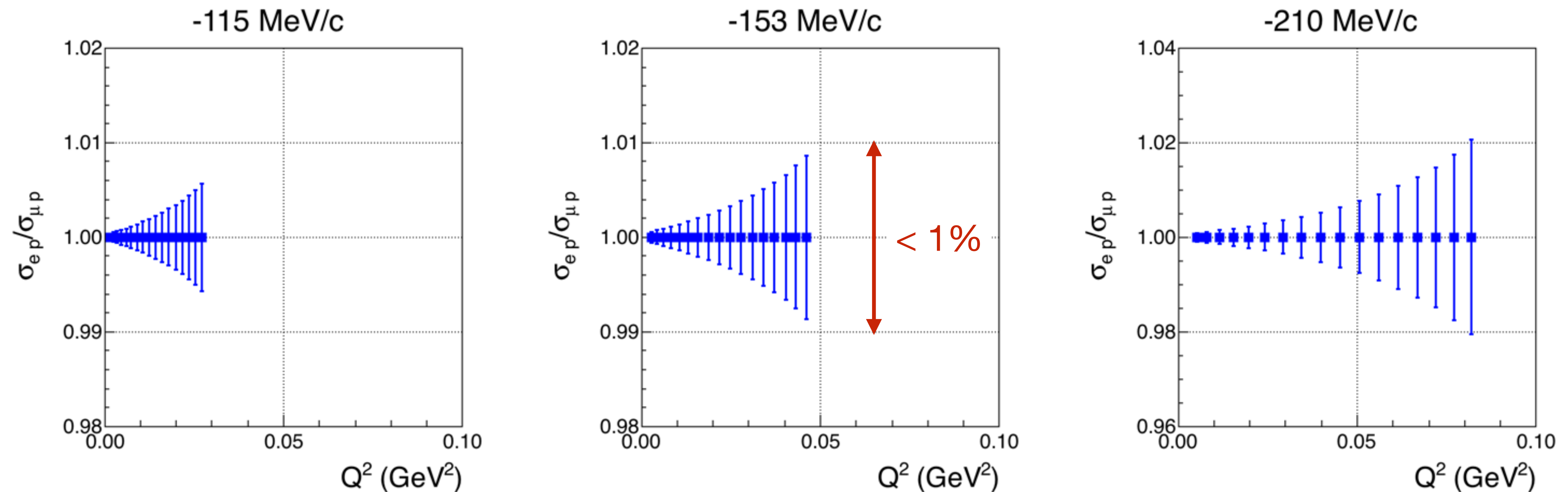
↑
sign change with lepton-charge

$$\frac{\sigma_{e^+p}}{\sigma_{e^-p}} = 1 + 4 \frac{\Re\{\mathcal{M}_{1\gamma}^\dagger \mathcal{M}_{2\gamma}\}}{|\mathcal{M}_{1\gamma}|^2}$$



MUSE directly compares ep to μp cross-sections

Projected relative statistical uncertainties in the ratio of **ep to μp** elastic **cross sections**.
Estimated systematics $\approx 0.5\%$.



The relative statistical uncertainties in the **form factors** are half as large.

MUSE is an unusual scattering experiment

Measure e^\pm and μ^\pm elastic scattering off a liquid hydrogen target.

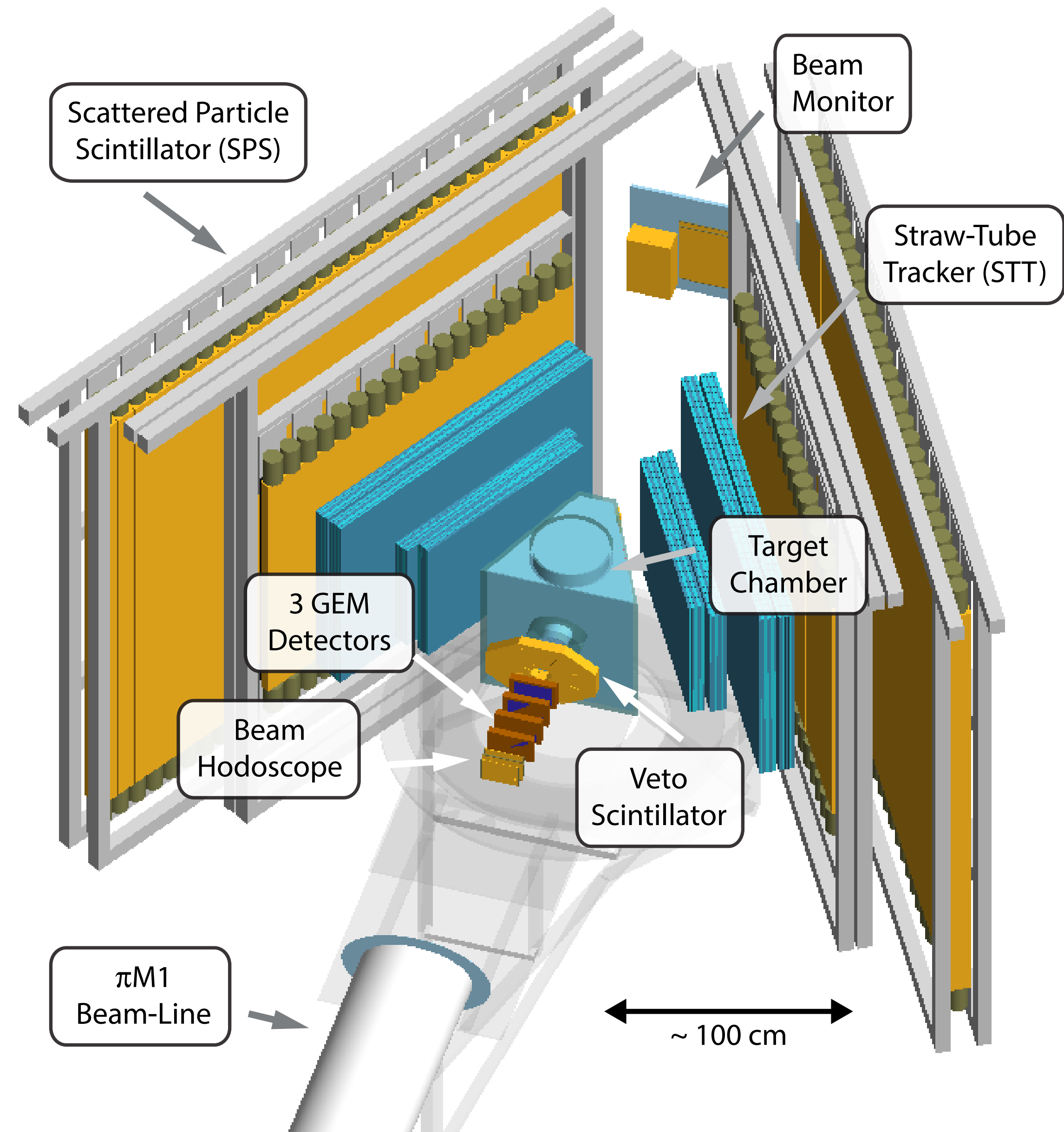
$$20^\circ < \theta < 100^\circ$$

$$-45^\circ < \phi < 45^\circ$$

$$p = 115, 161, 210 \text{ MeV}/c$$

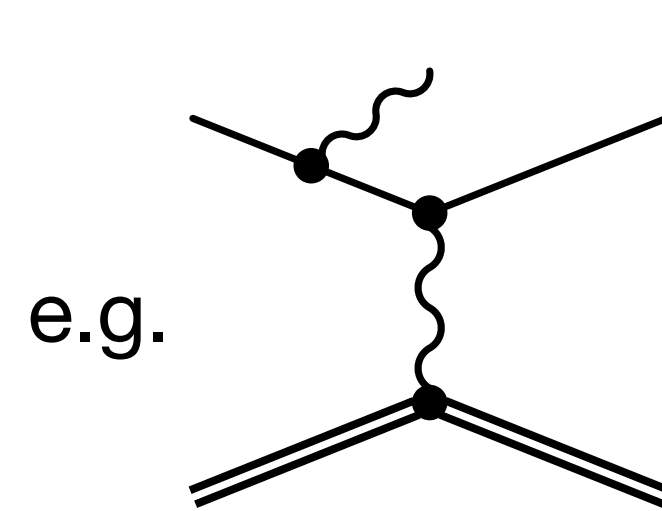
Challenges

- Secondary beam: identifying and tracking beam particles to target,
- Low beam flux: large angle, non-magnetic spectrometer,
- Background: e.g., Møller scattering and muon decay in flight.



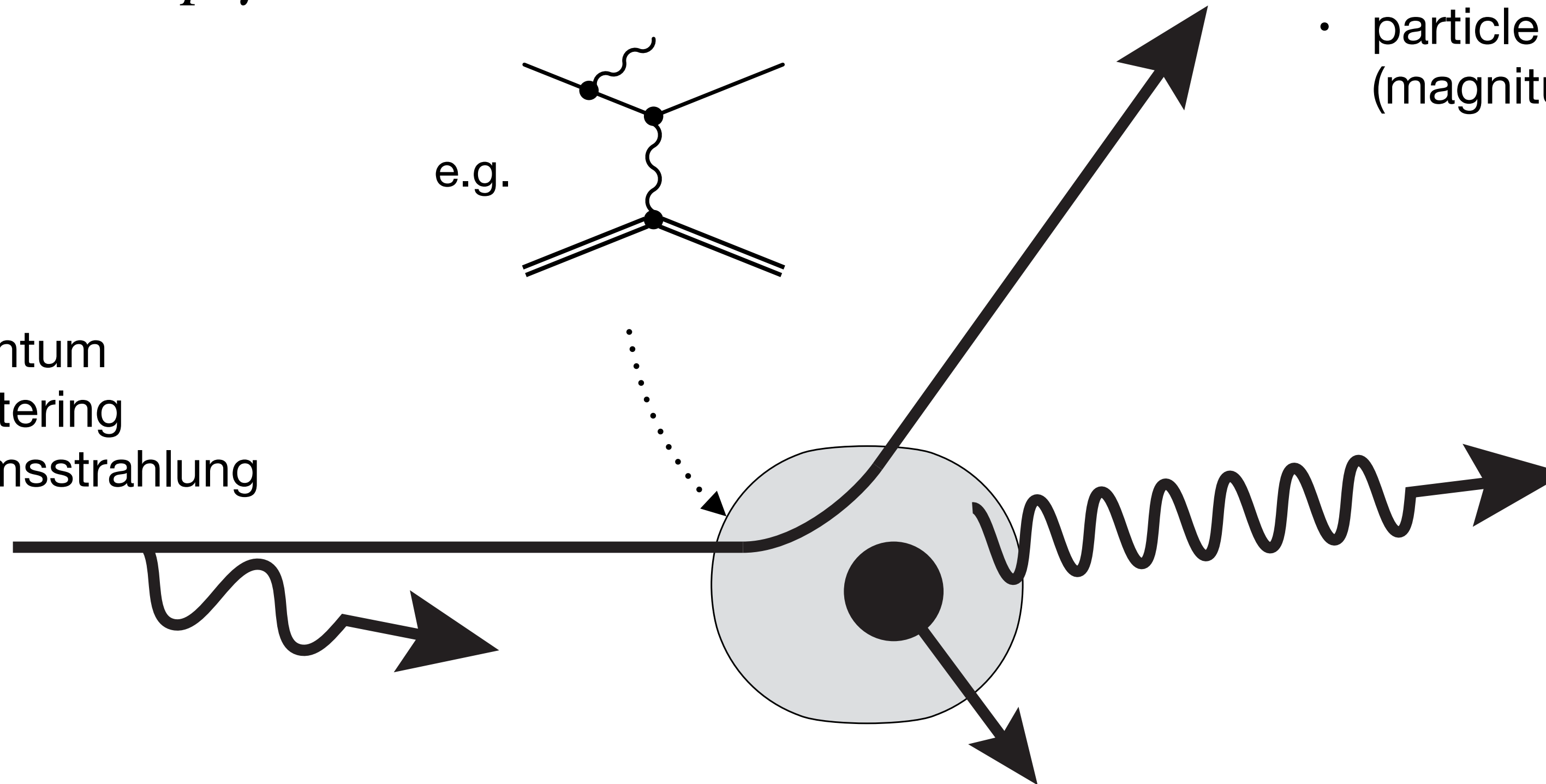
The size of the radiative corrections depends on the detector properties and event selection

$$\ell^{\pm} p \rightarrow \ell'^{\pm} p' \gamma$$



Incident lepton

- beam momentum
- multiple scattering
- external Bremsstrahlung



Scattered lepton

- angular acceptance
- particle momentum (magnitude not precisely measured)

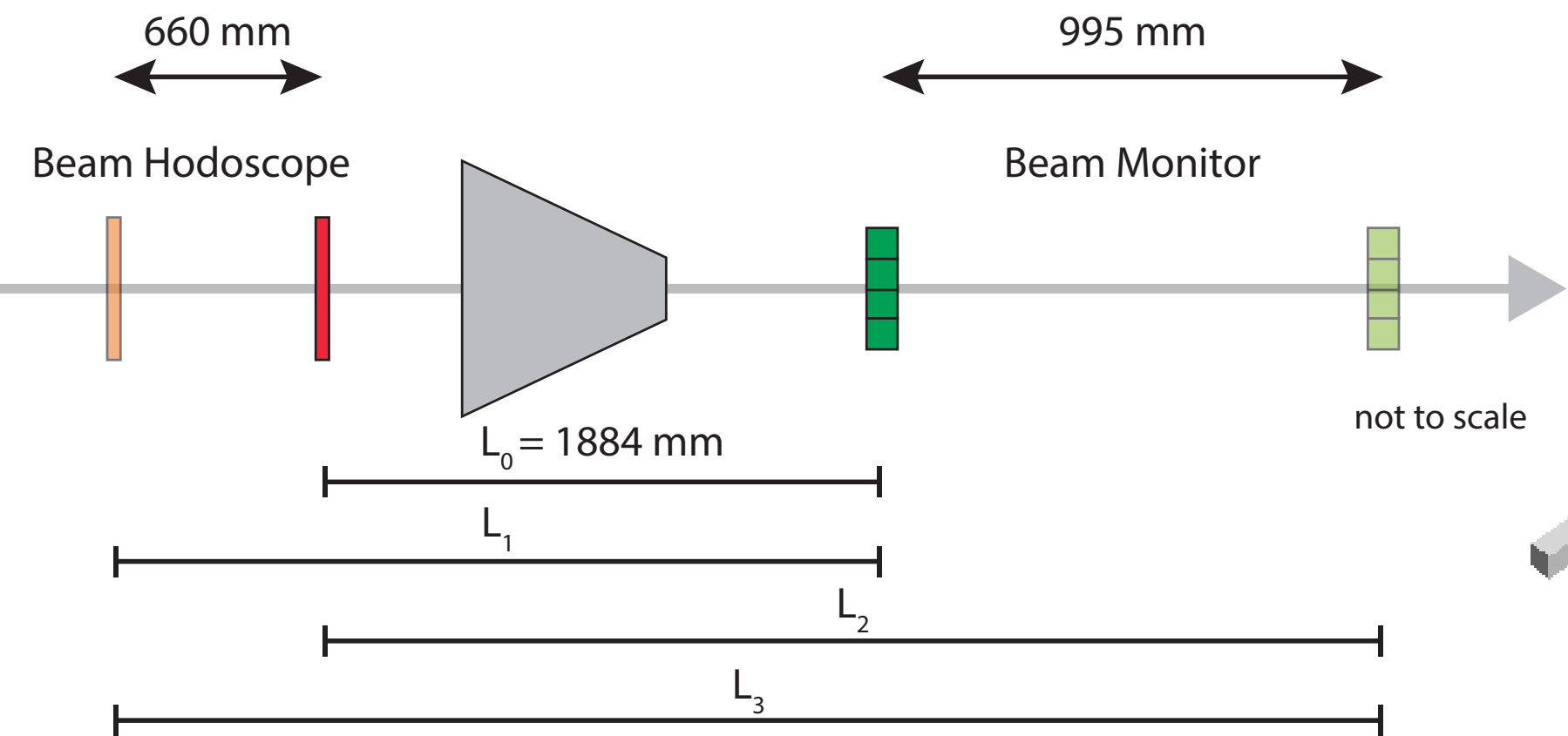
Internal Bremsstrahlung

e.g., initial-state radiation

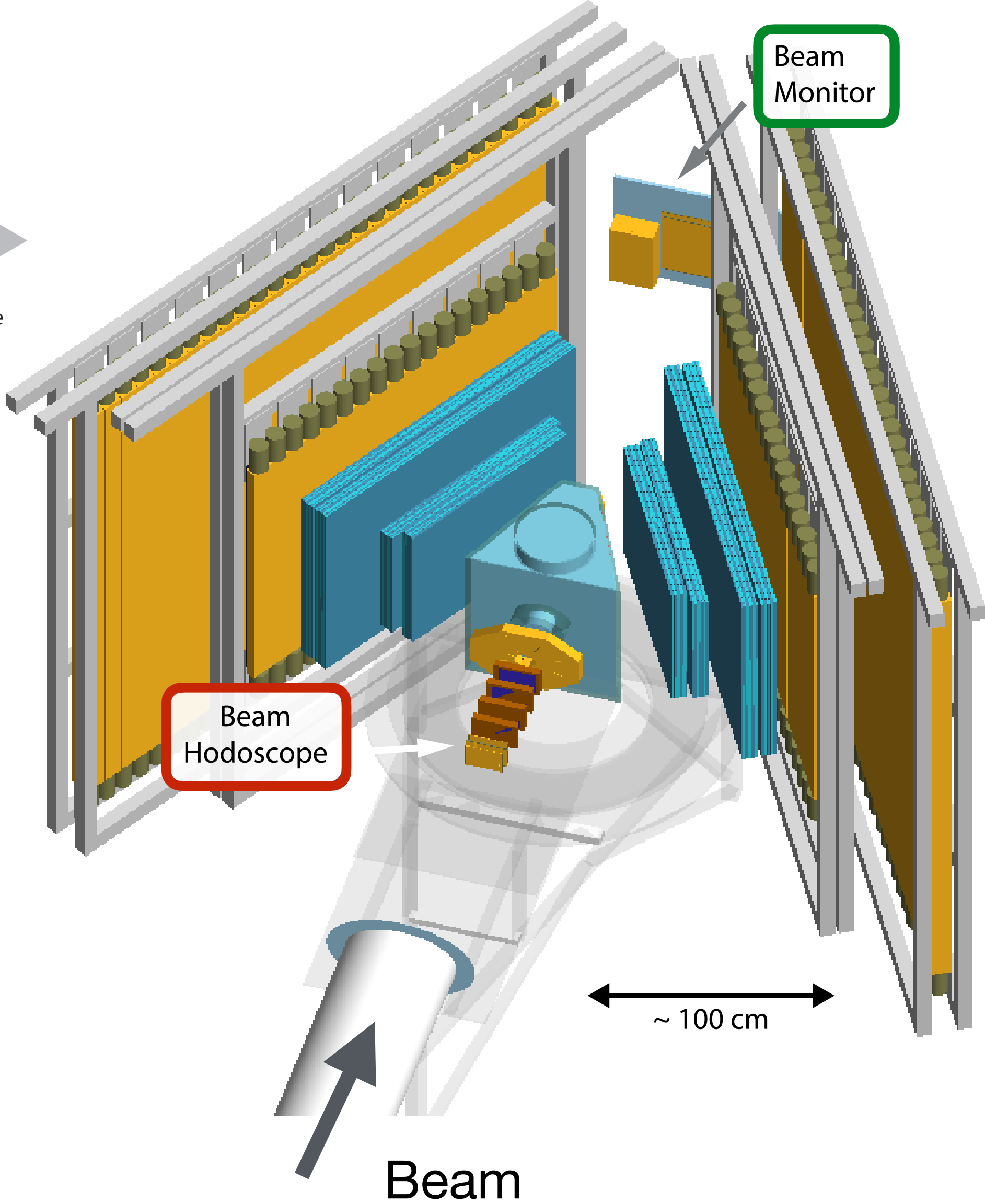
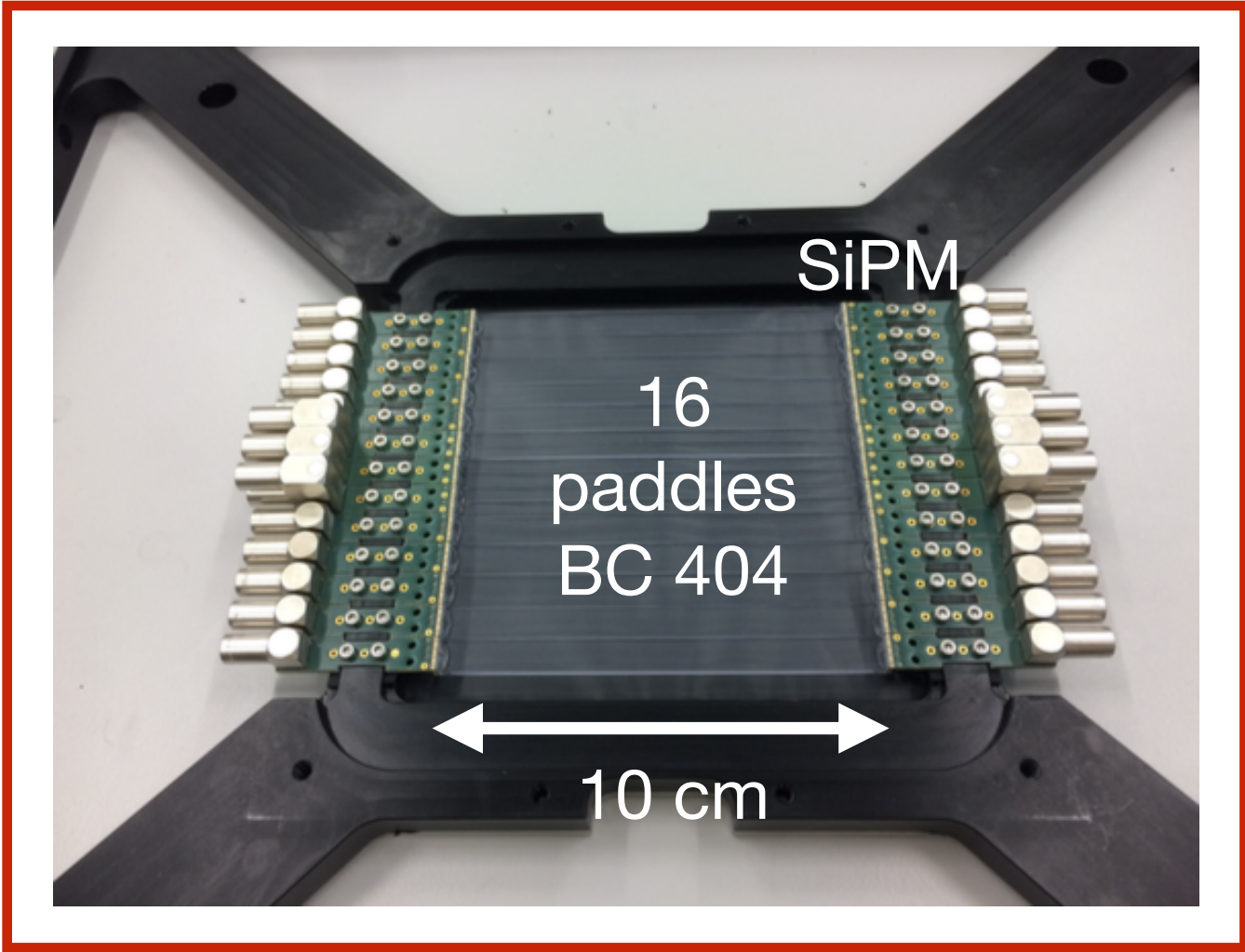
Recoiling proton

remains unobserved

MUSE detector system for TOF measurements



Beam hodoscope
planes C & D (Rutgers)



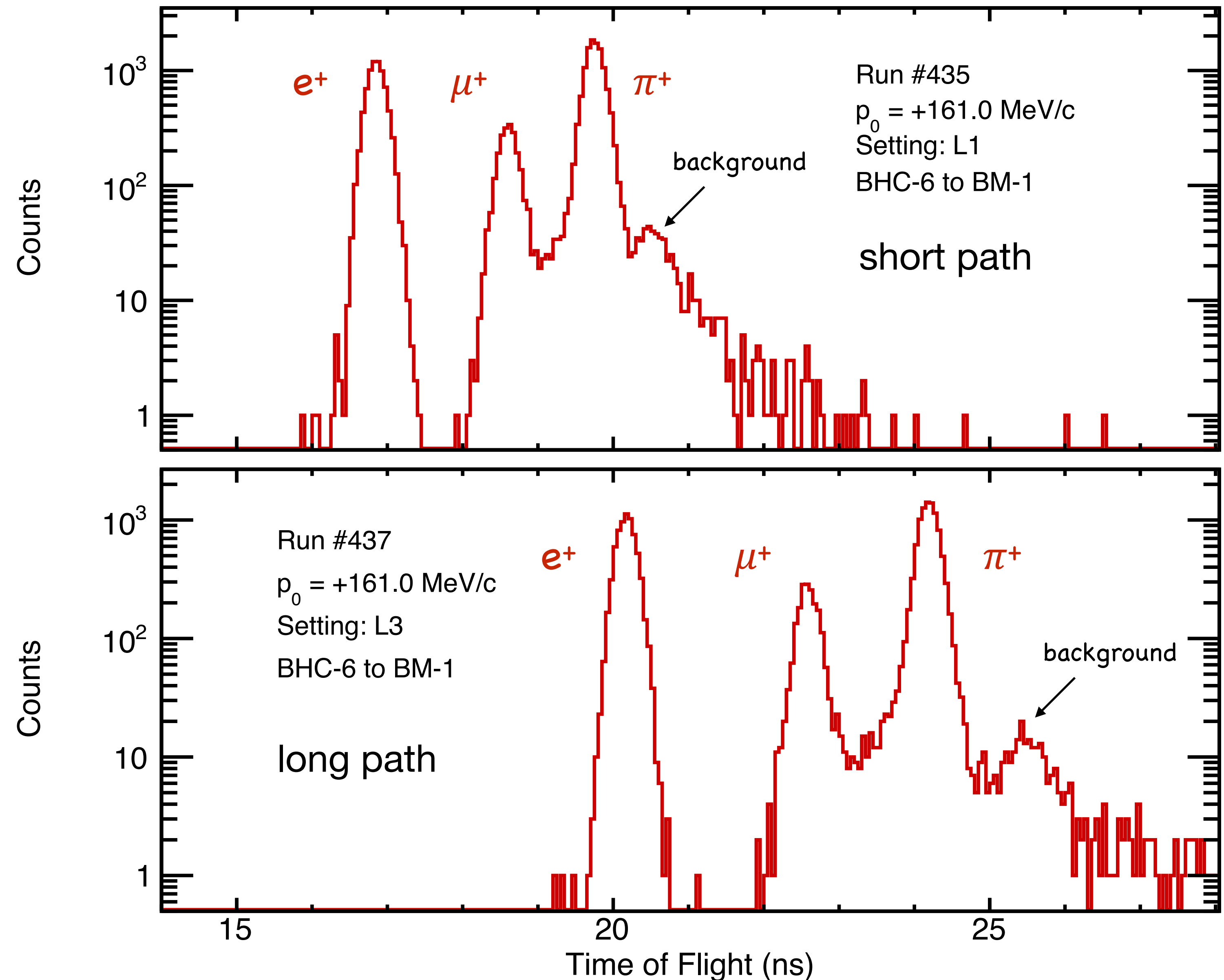
Beam monitor
SC bars (UofSC)



Examples of two experimental time-of-flight distributions

The agreement of the measured and calculated π M1 channel momenta is at the desired 0.2 – 0.3% level.

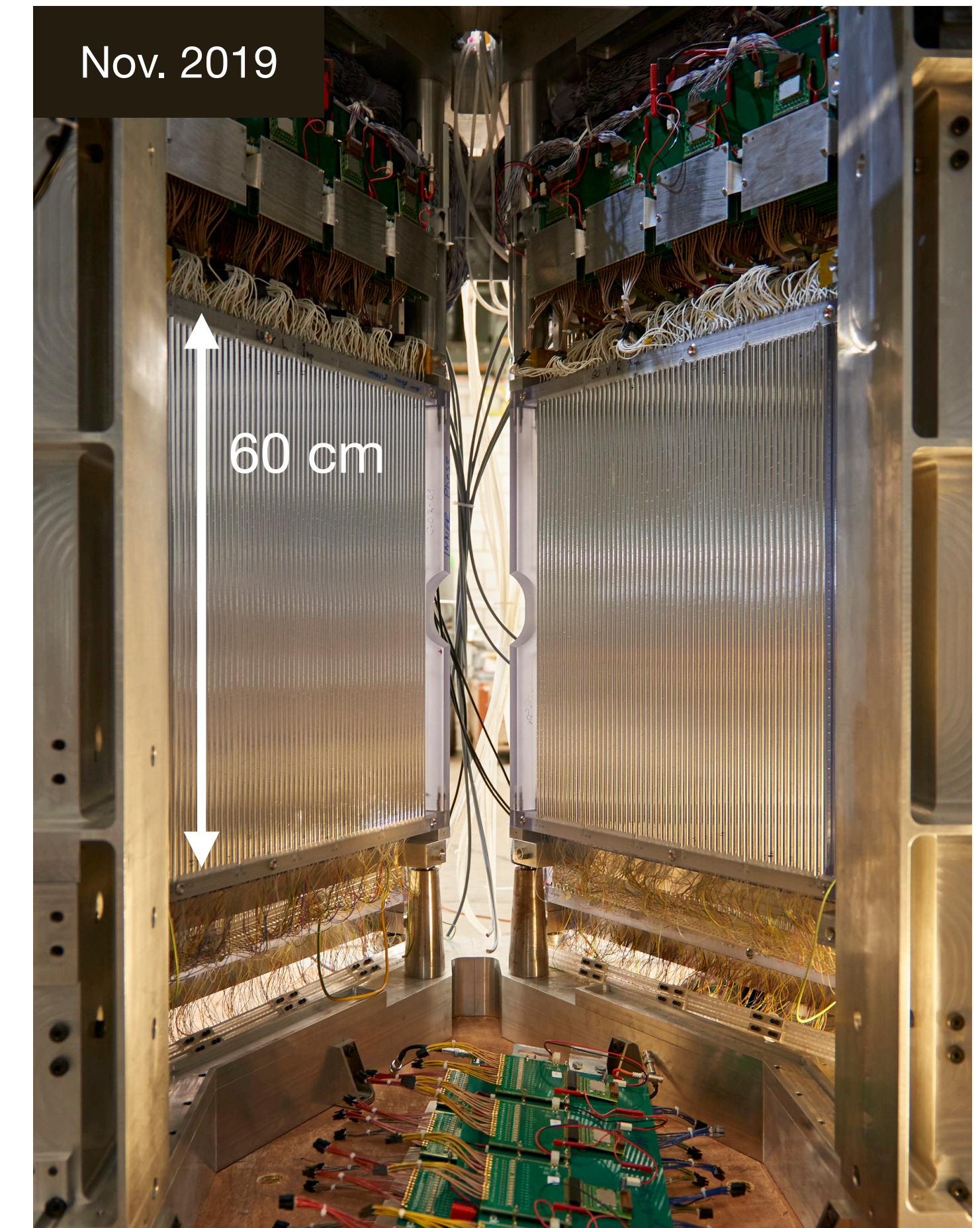
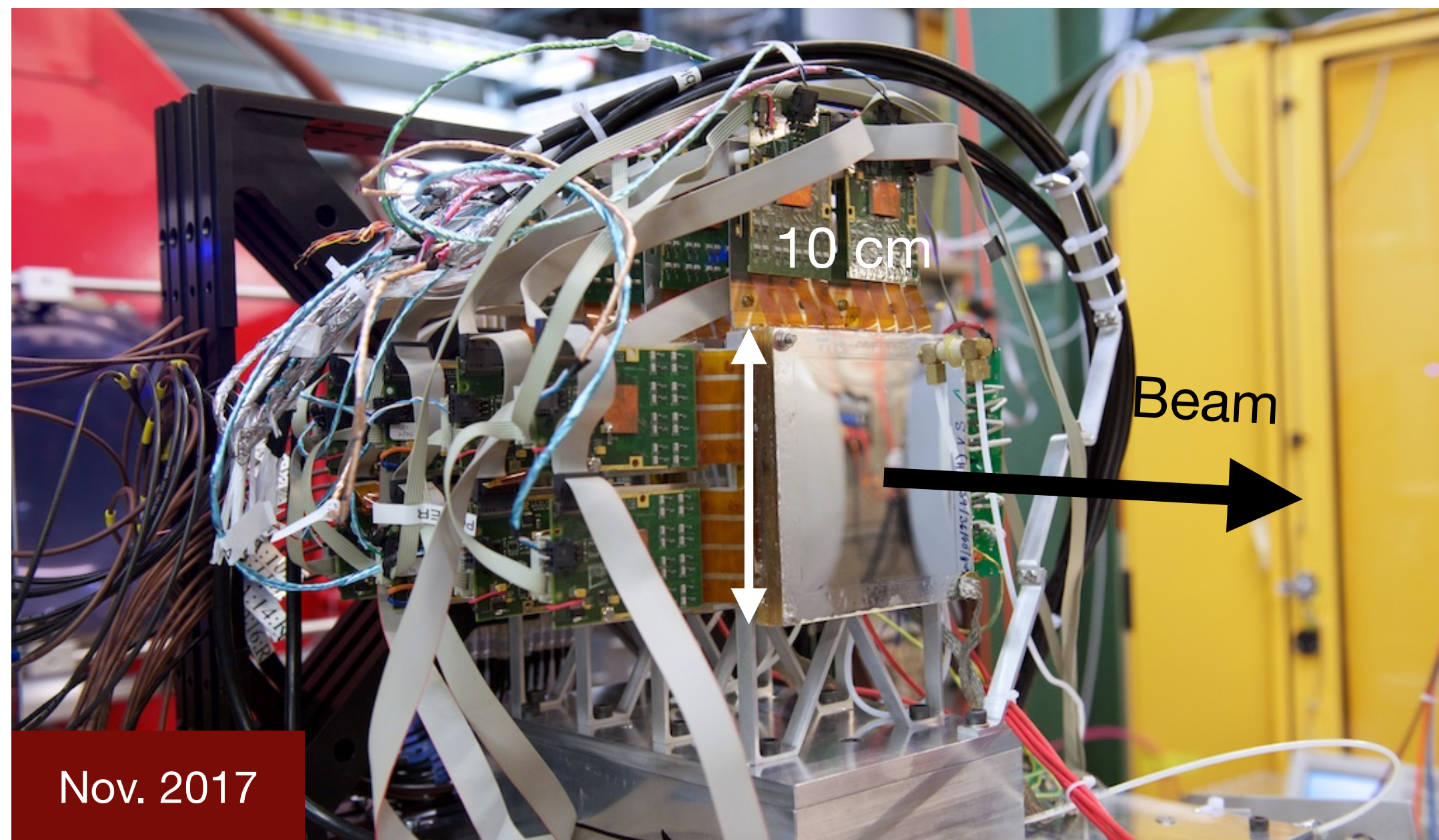
$$\sigma_{p_0} = 0.002 \cdot p_0$$



MUSE tracking detectors

GEM detectors (Hampton Univ.)

- Set of three GEM detectors built for & run in OLYMPUS.
- Measure trajectories into the target to reconstruct the scattering kinematics.

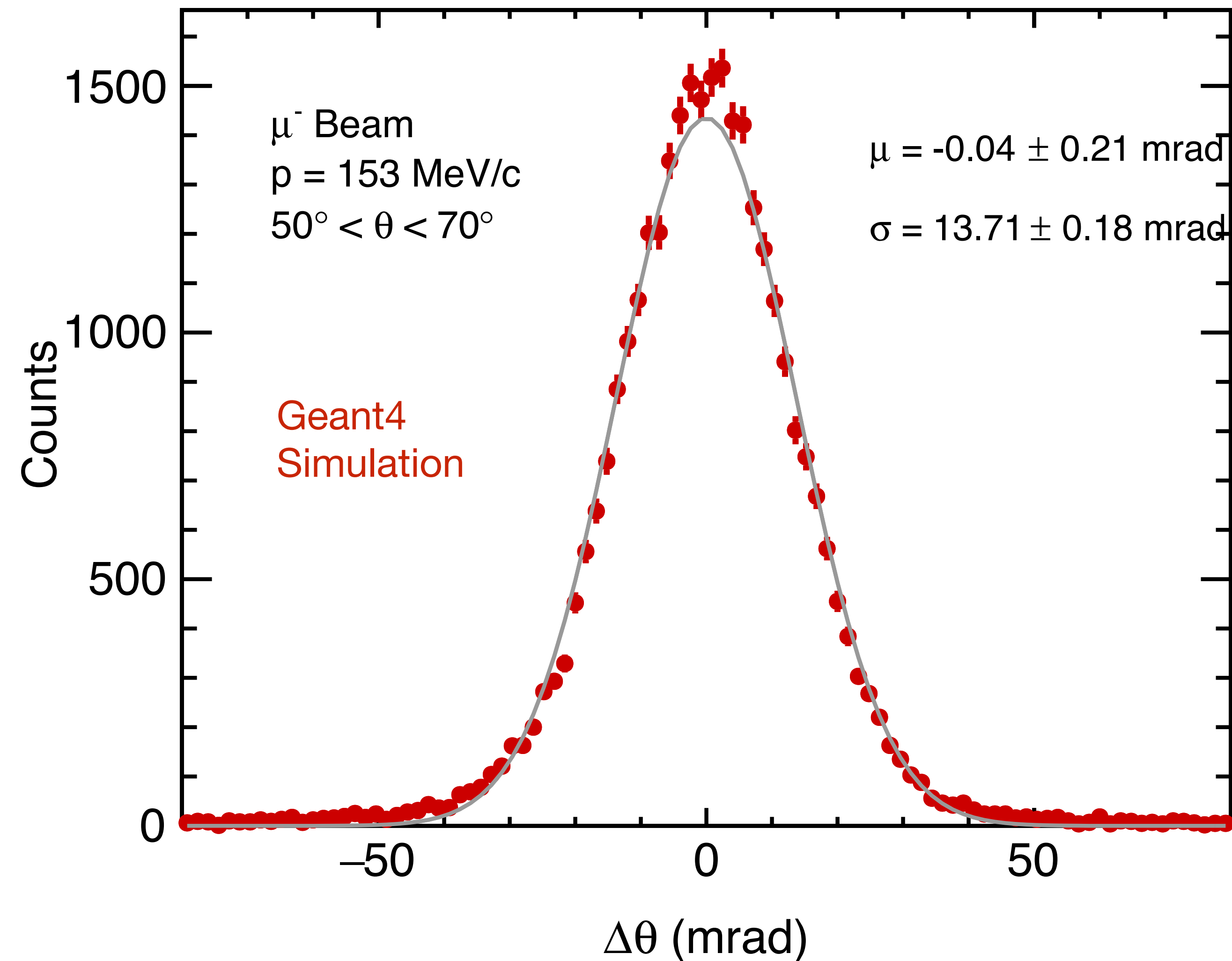


Straw-tube tracker

(Hebrew University of Jerusalem + Temple)

- Two STT chambers with 5 vertical and 5 horizontal planes each (3000 straws total).
- The Straw Tube Tracker provides high-resolution and high-efficiency tracking of the scattered particles from the target.

Reconstruction of scattering angle



Position resolution:
GEM 70 μm and STT 120 μm .

Full Geant4 simulation including
detector material and target.

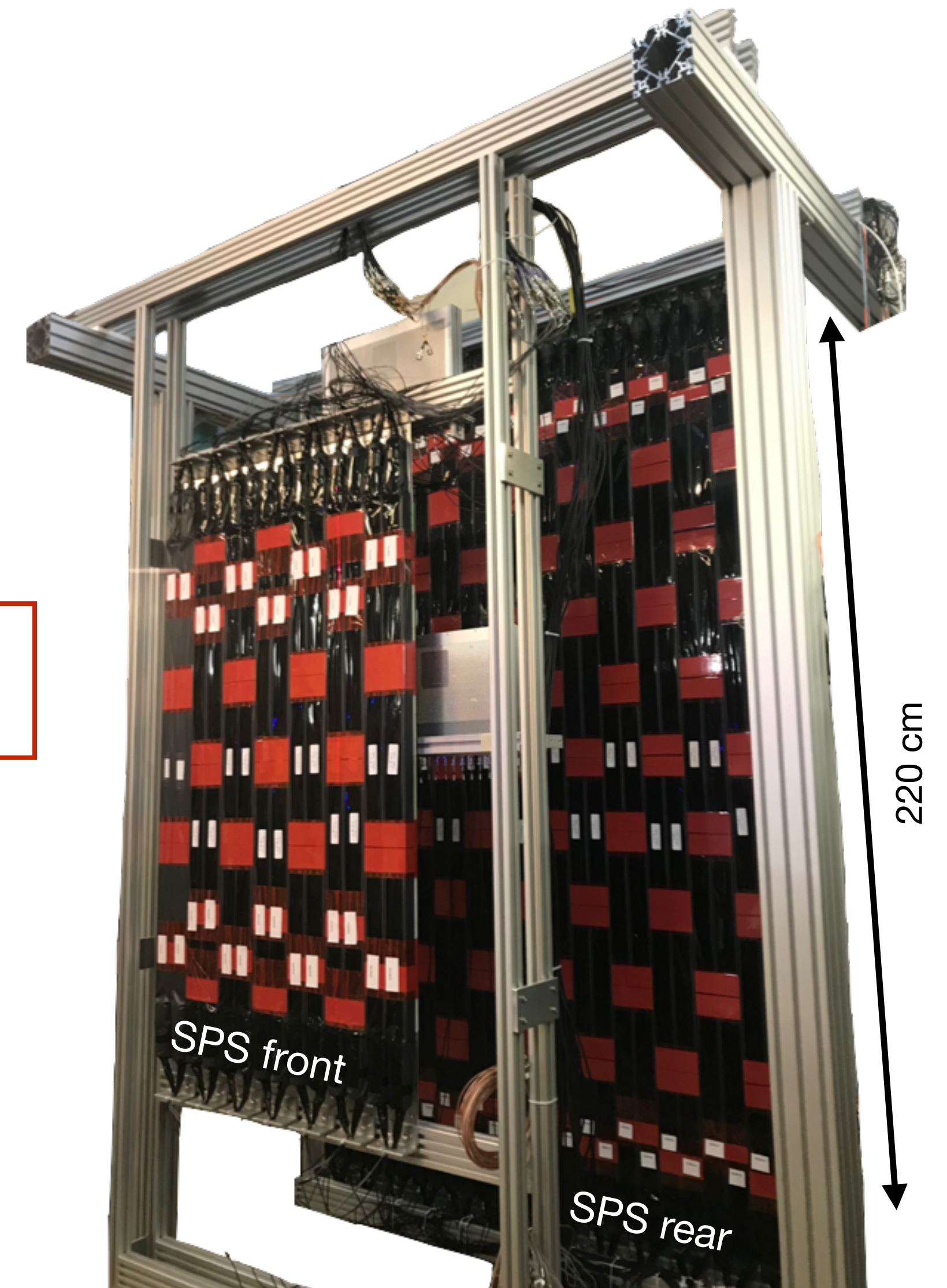
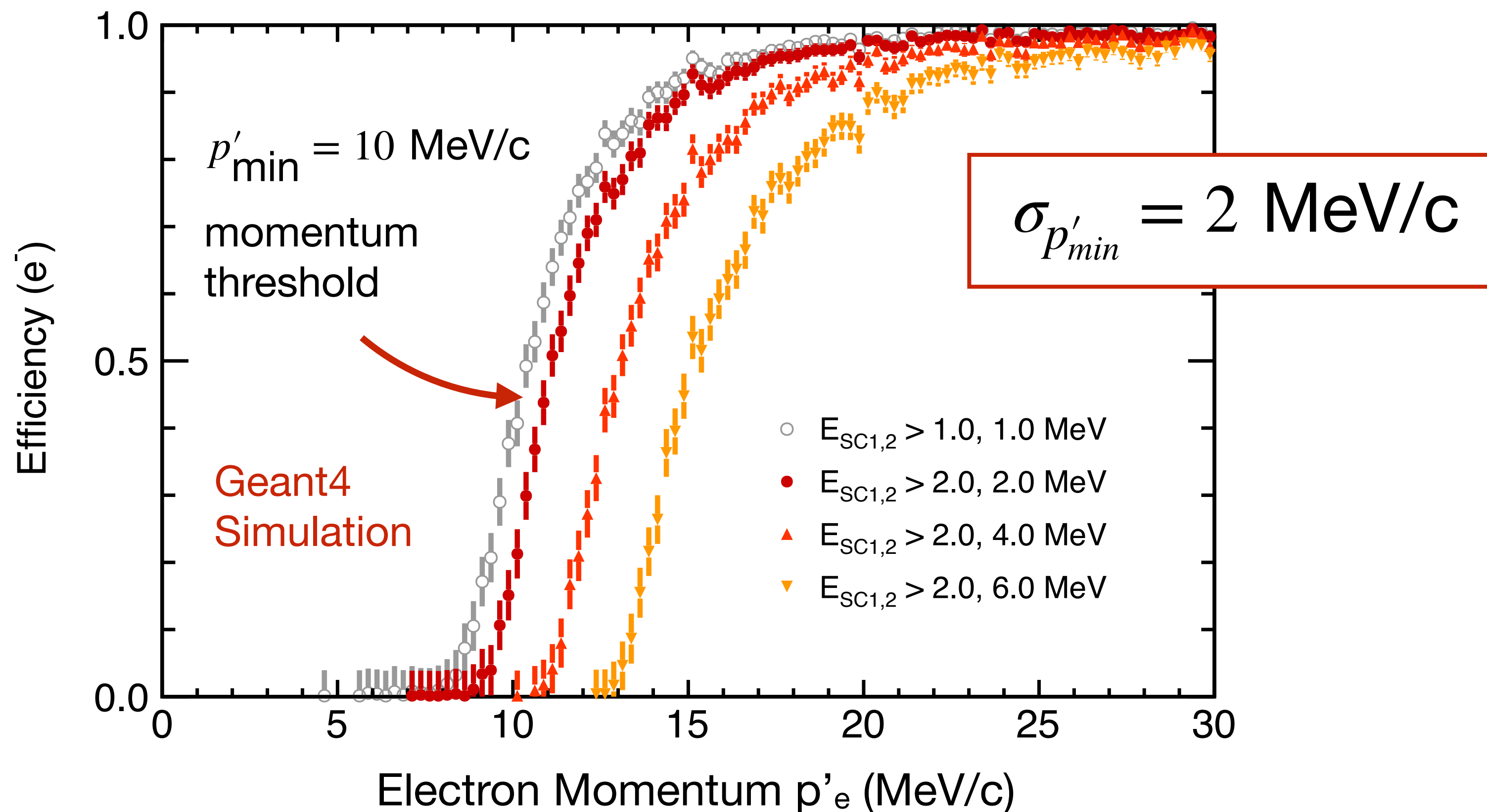
Scattering-angular resolution is
dominated by multiple scattering
and $\leq 20 \text{ mrad}$.

$$\sigma_\theta = 20 \text{ mrad}$$

Detection of scattered lepton in SPS

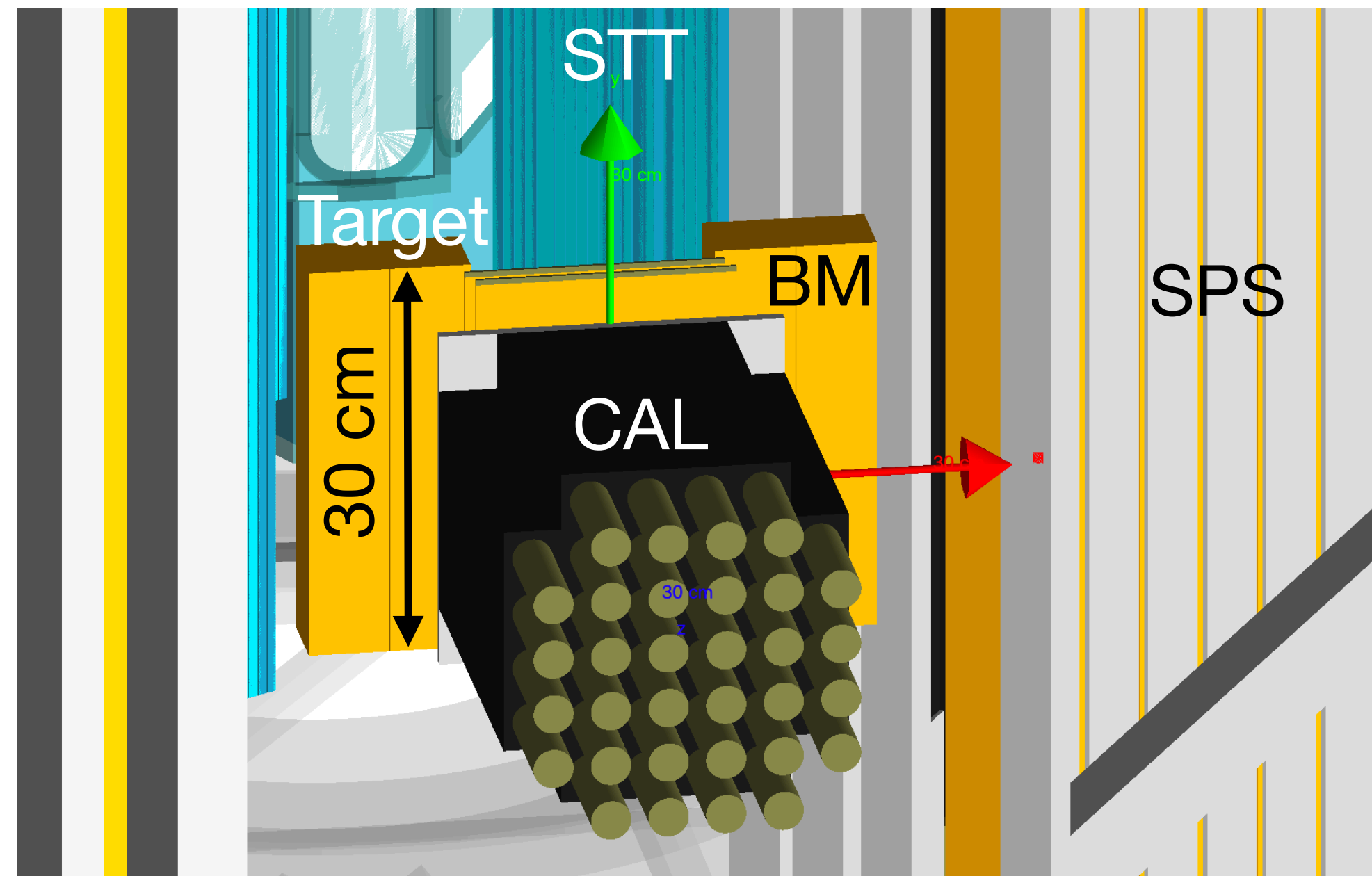
Fast and efficient **scattered-particle scintillators (SPS)**
(University of South Carolina) provide event trigger and particle ID

- Front wall: 18 bars (6 cm x 3 cm x 120 cm), $\sigma_t < 50$ ps
- Rear wall: 28 bars (6 cm x 6 cm x 220 cm), $\sigma_t < 60$ ps



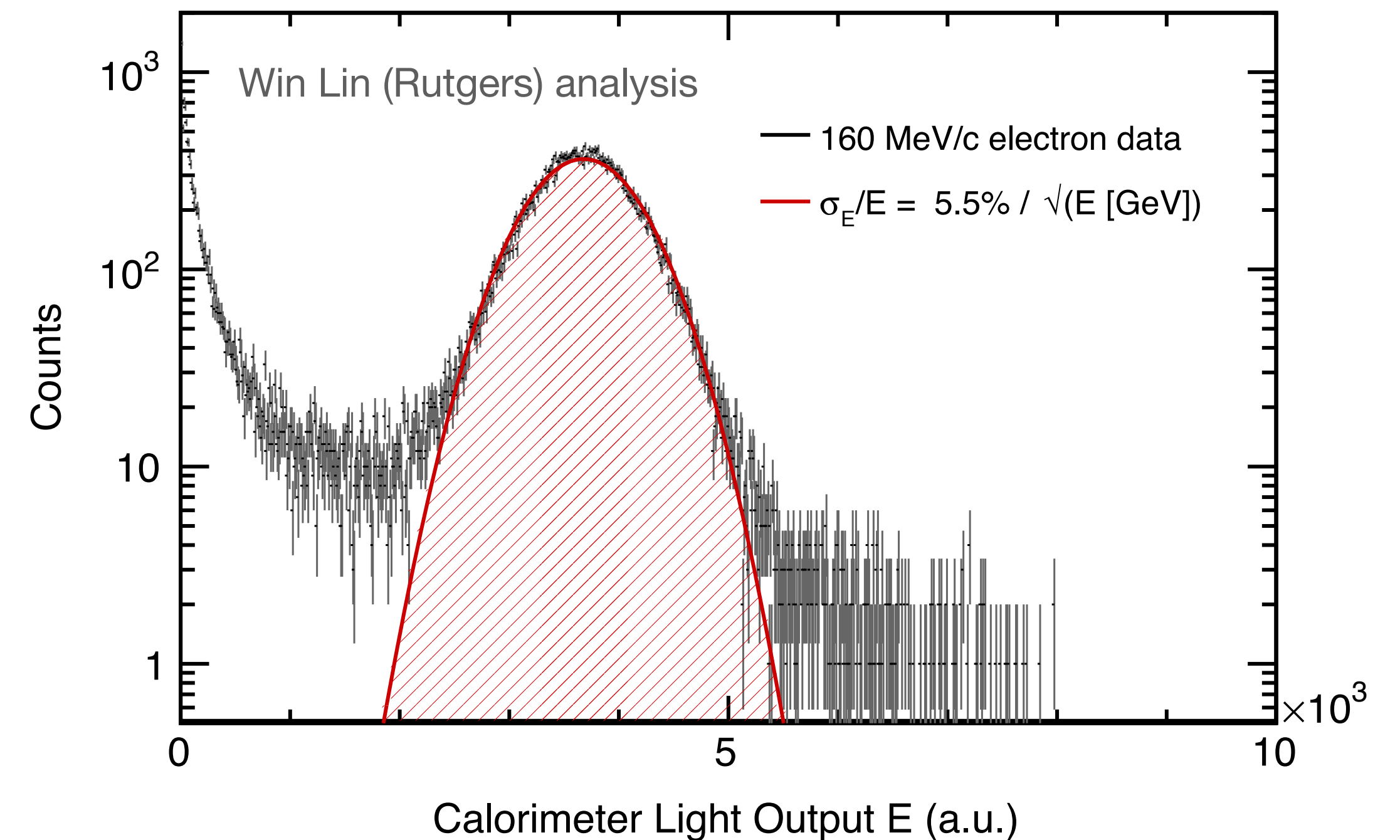
Calorimeter

Beam line view from downstream side of MUSE



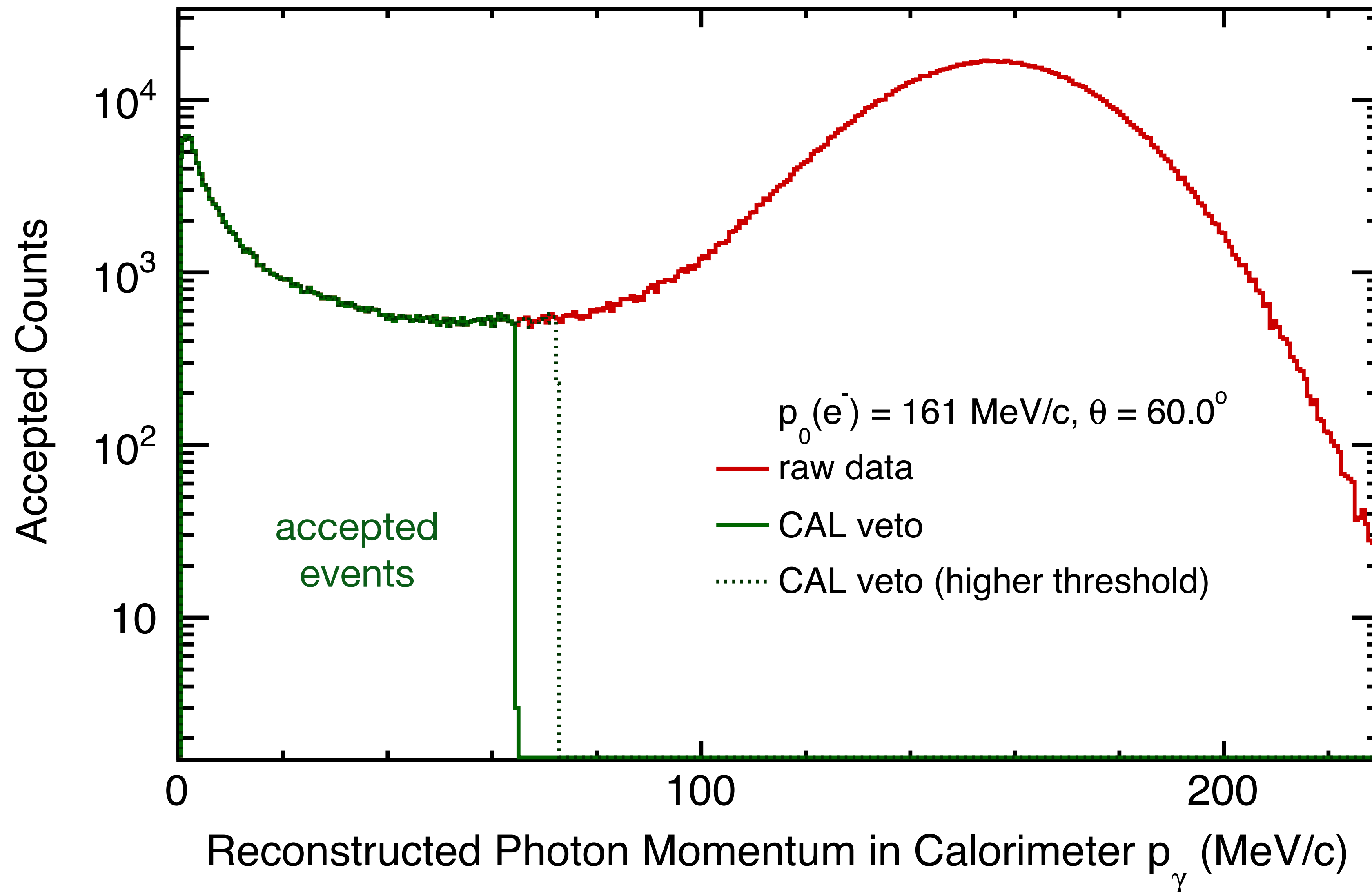
Calorimeter in beamline downstream of BM
32 lead-glas crystals (4 cm x 4 cm x 30 cm)

Preliminary performance tests
Sum of light output of set of nine calorimeter blocks



$$\frac{\sigma_E}{E} = \frac{5\%}{\sqrt{E \text{ (GeV)}}}$$

Calorimeter can be used to suppress initial-state bremsstrahlung



Simulated
bremsstrahlung
spectrum, folded with
expected detector
resolution

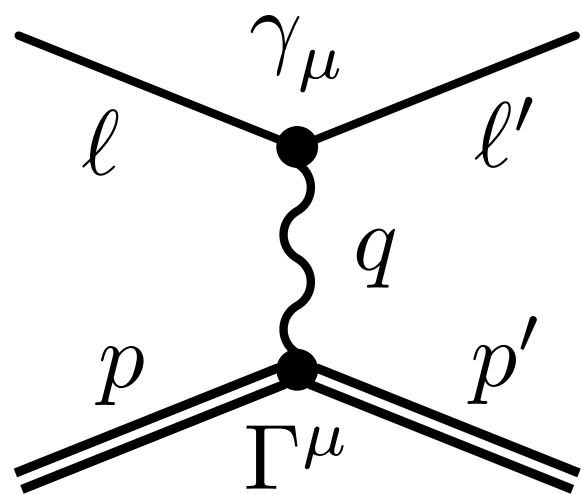
$$\sigma_{E_\gamma} \approx 10 \text{ MeV}$$

Event-Generator Requirements

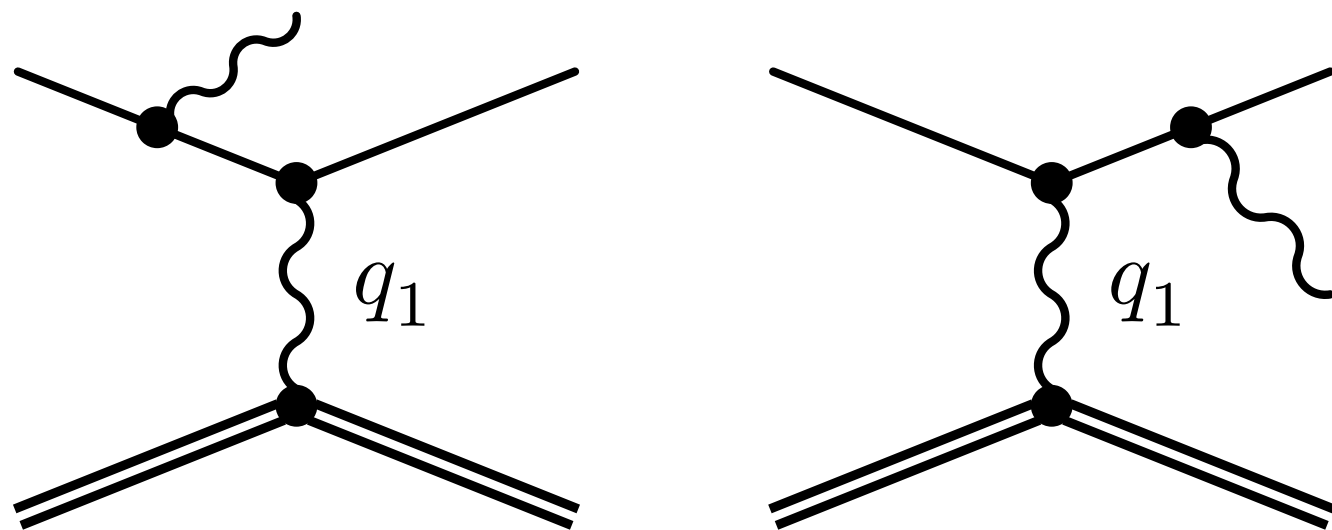
- Include emission of **hard radiated photon**, beyond soft-photon approximation.
- Include the **mass of lepton**:
 - Not using approximation $-q^2 \gg m^2$.
 - Accurate calculation of the event kinematics.
- Suitable for Monte Carlo simulations
- Models
 - **ESEPP** (Gramolin et al.)
 - Olympus / Darklight
 - Heavy baryon chiral perturbation theory (F. Myhrer and collaborators)
 - ELRADGEN (A. Afanasev et al.)
 - ...

Elastic Scattering of Electrons and Positrons on Protons (ESEPP)

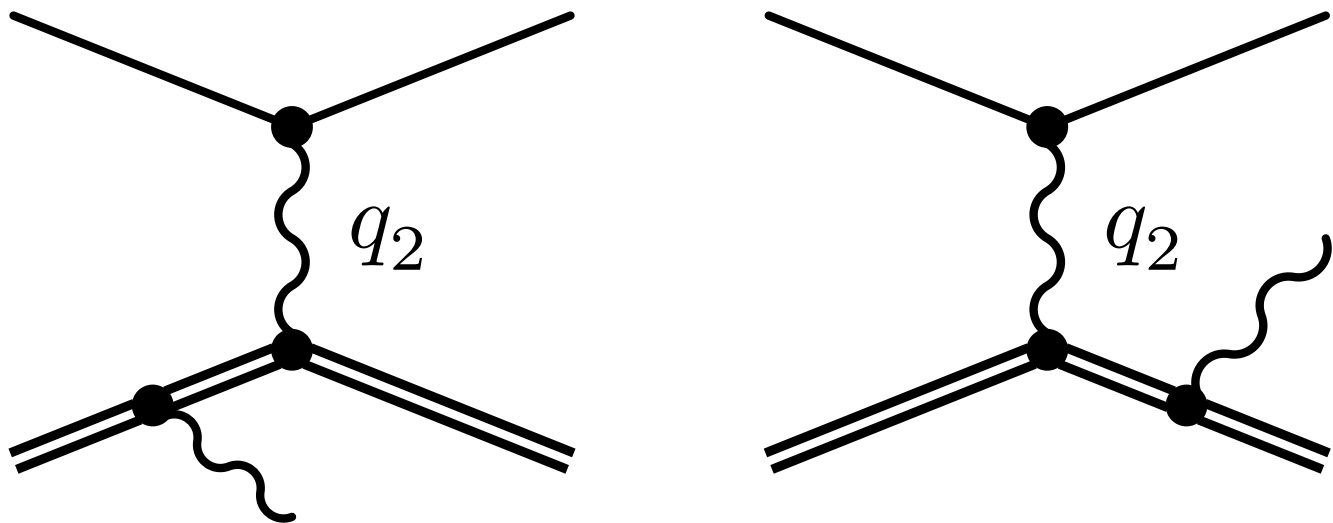
first Born approximation



first-order bremsstrahlung process

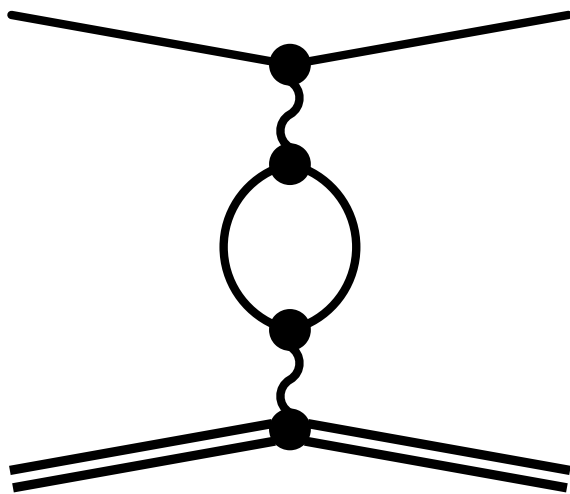


$$l^{\pm}p \rightarrow l'^{\pm}p'\gamma$$

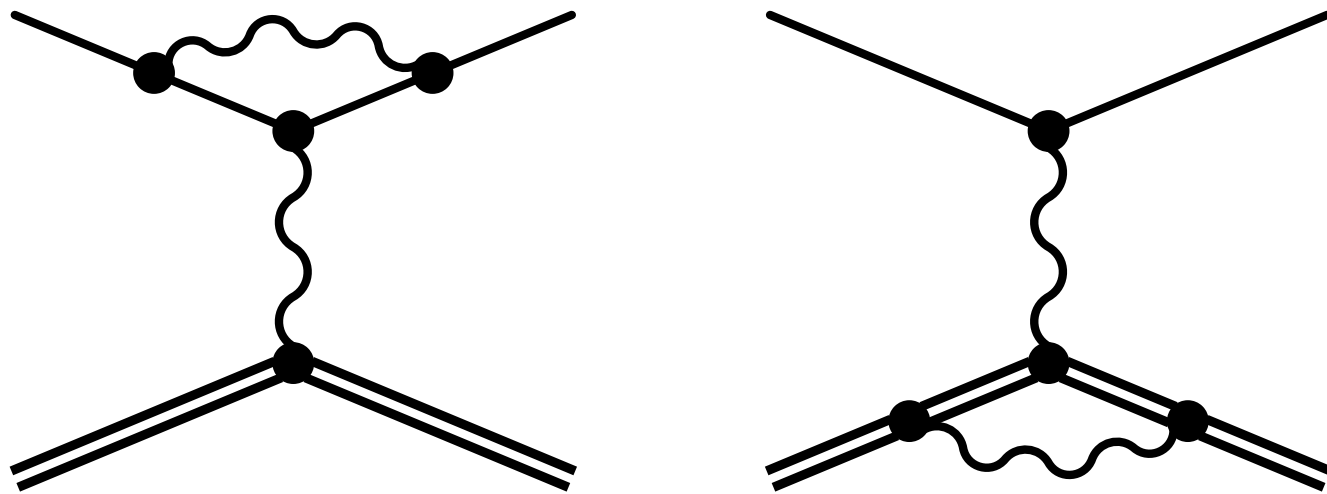


virtual-photon corrections

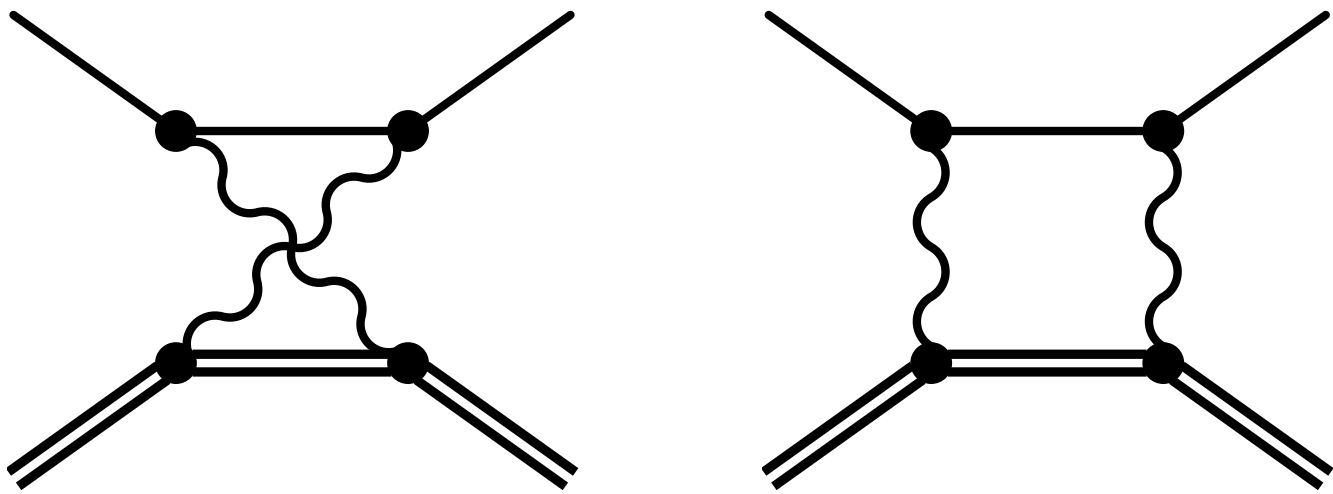
vacuum polarization



lepton/proton vertex corrections



TPE corrections



Lepton-charge dependence

Cross section of charged lepton scattering on protons:

$$\begin{aligned}
 \sigma(\ell^\pm p) \propto & |\mathcal{M}_{\text{Born}}|^2 + \\
 & 2\Re[\mathcal{M}_{\text{Born}}^\dagger (\mathcal{M}_{\text{vac}} + \mathcal{M}_{\text{vert}}^\ell + \mathcal{M}_{\text{vert}}^p)] + \\
 & \boxed{2\Re[\mathcal{M}_{\text{Born}}^\dagger (\mathcal{M}_{\text{box}} + \mathcal{M}_{\text{xbox}})] +} \\
 & |\mathcal{M}_{\text{brems}}^{\text{li}} + \mathcal{M}_{\text{brems}}^{\text{lf}}|^2 + |\mathcal{M}_{\text{brems}}^{\text{pi}} + \mathcal{M}_{\text{brems}}^{\text{pf}}|^2 + \\
 & \boxed{2\Re[(\mathcal{M}_{\text{brems}}^{\text{li}} + \mathcal{M}_{\text{brems}}^{\text{lf}})^\dagger (\mathcal{M}_{\text{brems}}^{\text{pi}} + \mathcal{M}_{\text{brems}}^{\text{pf}})] + \mathcal{O}(\alpha^4)}
 \end{aligned}$$

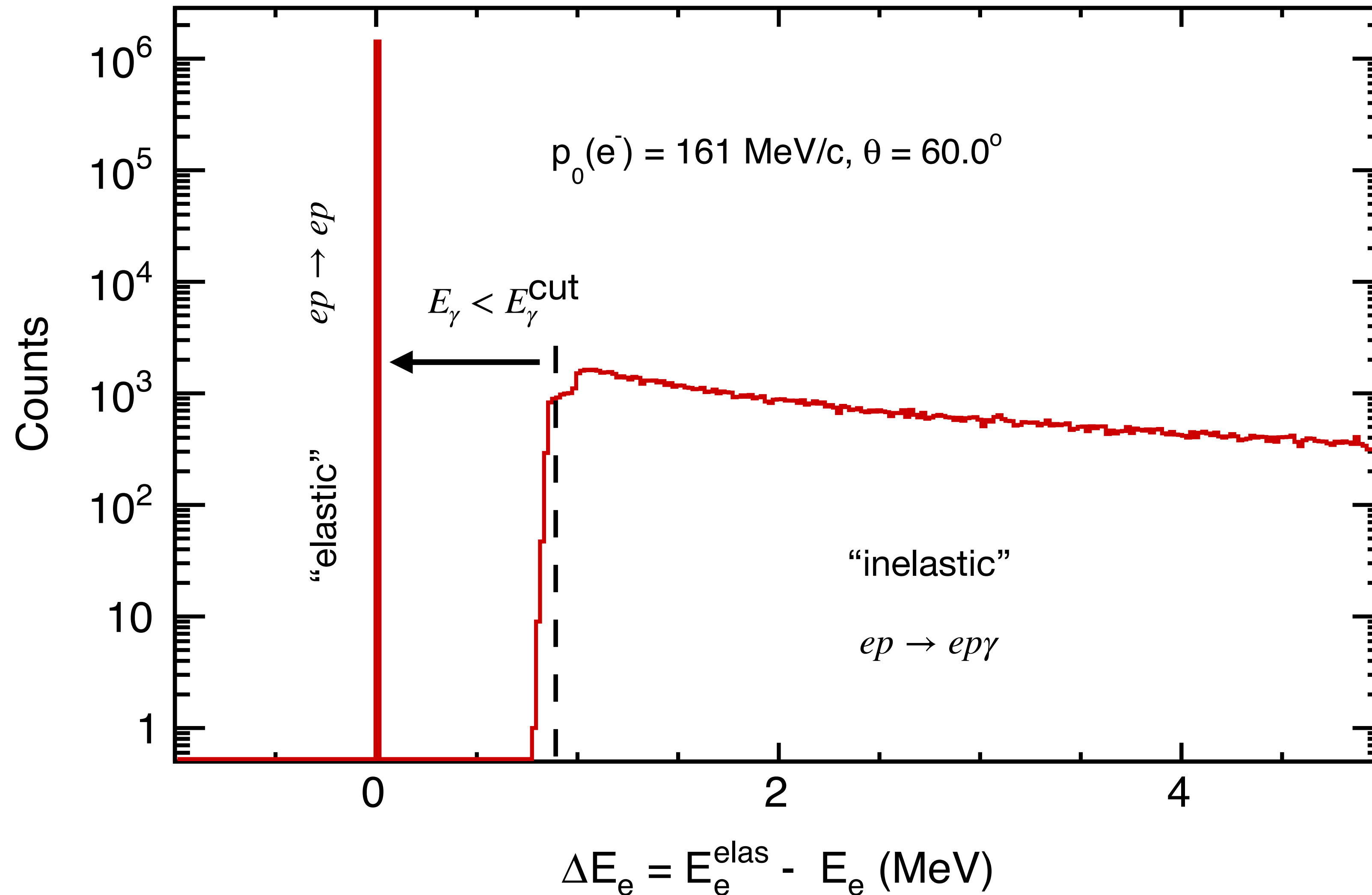
Charge-odd terms:

Interference TPE term

Interference bremsstrahlung term

The interference-TPE term and the interference-bremsstrahlung terms change sign depending on the sign of the lepton's charge.

The ESEPP event generator



- ESEPP generates unweighted events
- Two types of events: elastic (analytical integration) and inelastic (numerical integration)
- First-order bremsstrahlung is taken into account in both cases

Minimalistic MUSE simulation

Numerical integration of Bremsstrahlung cross-section

$$\frac{d\sigma}{d\Omega_l}(p'_{l,min}) = \int_{p'_l} \int_{\Omega_\gamma} \frac{d\sigma_{\text{brems}}}{d\Omega_l d\Omega_\gamma dp'_l} d\Omega_l dp'_l$$

Born cross-section

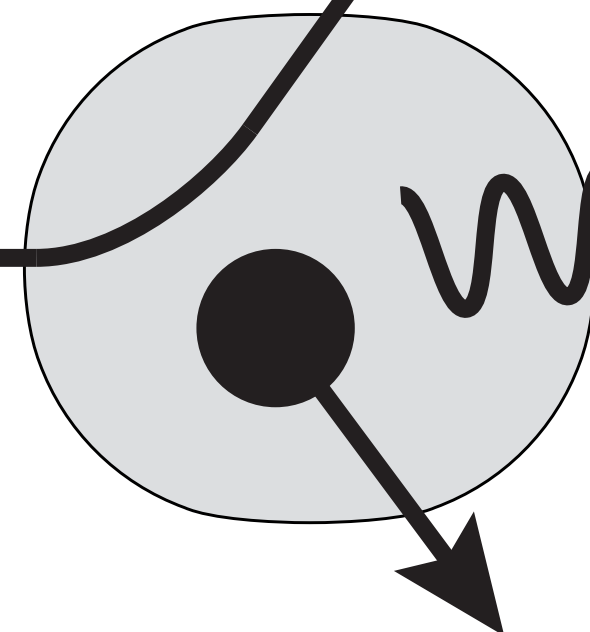
$$\frac{d\sigma}{d\Omega_l}(p'_{l,min}) = \frac{d\sigma_0}{d\Omega_l} \left[1 + \delta(p'_{l,min}) \right]$$

Radiative correction

$$\delta = \frac{d\sigma}{d\Omega_l} / \frac{d\sigma_0}{d\Omega_l} - 1$$

Fixed momentum
lepton beam

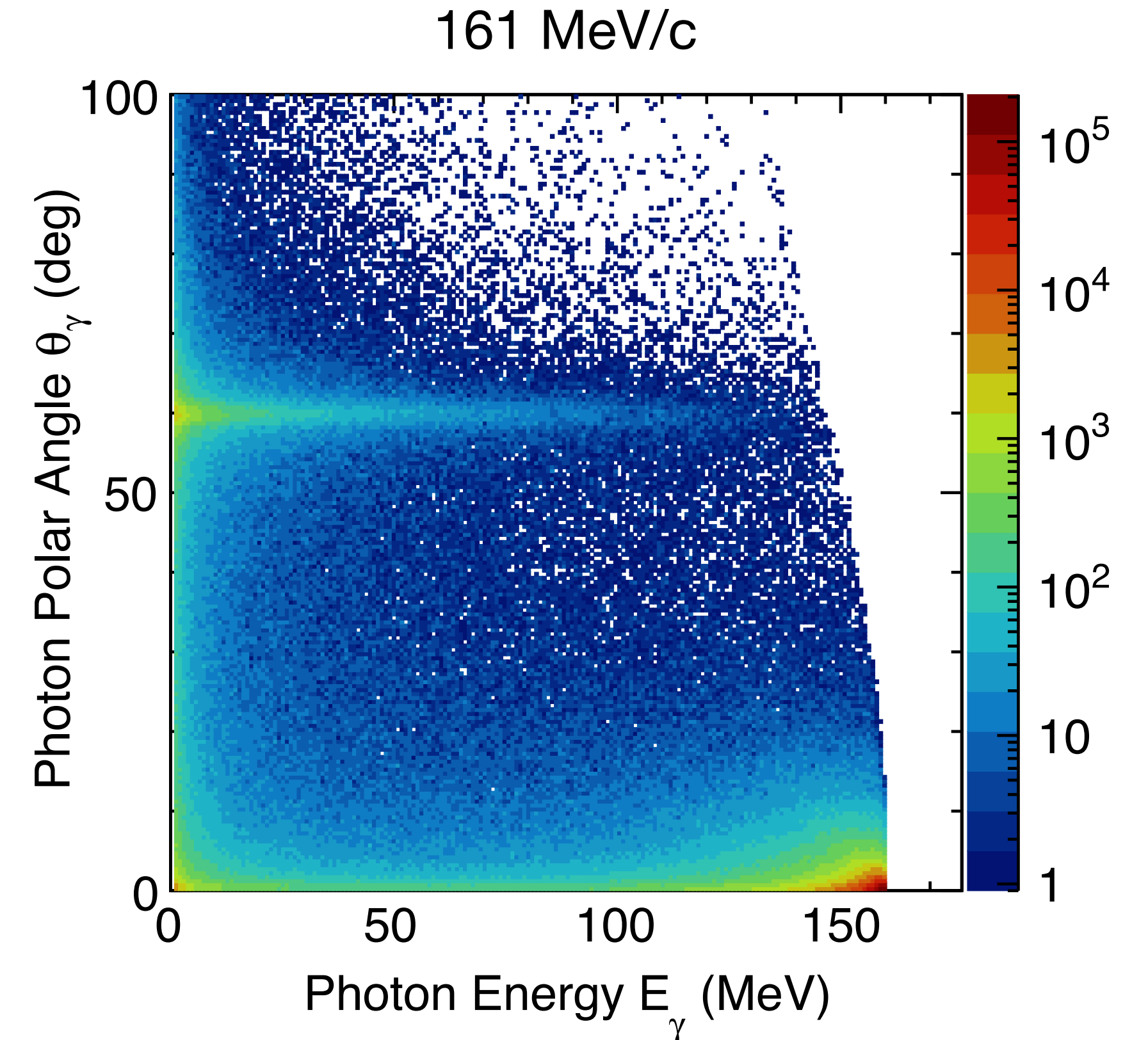
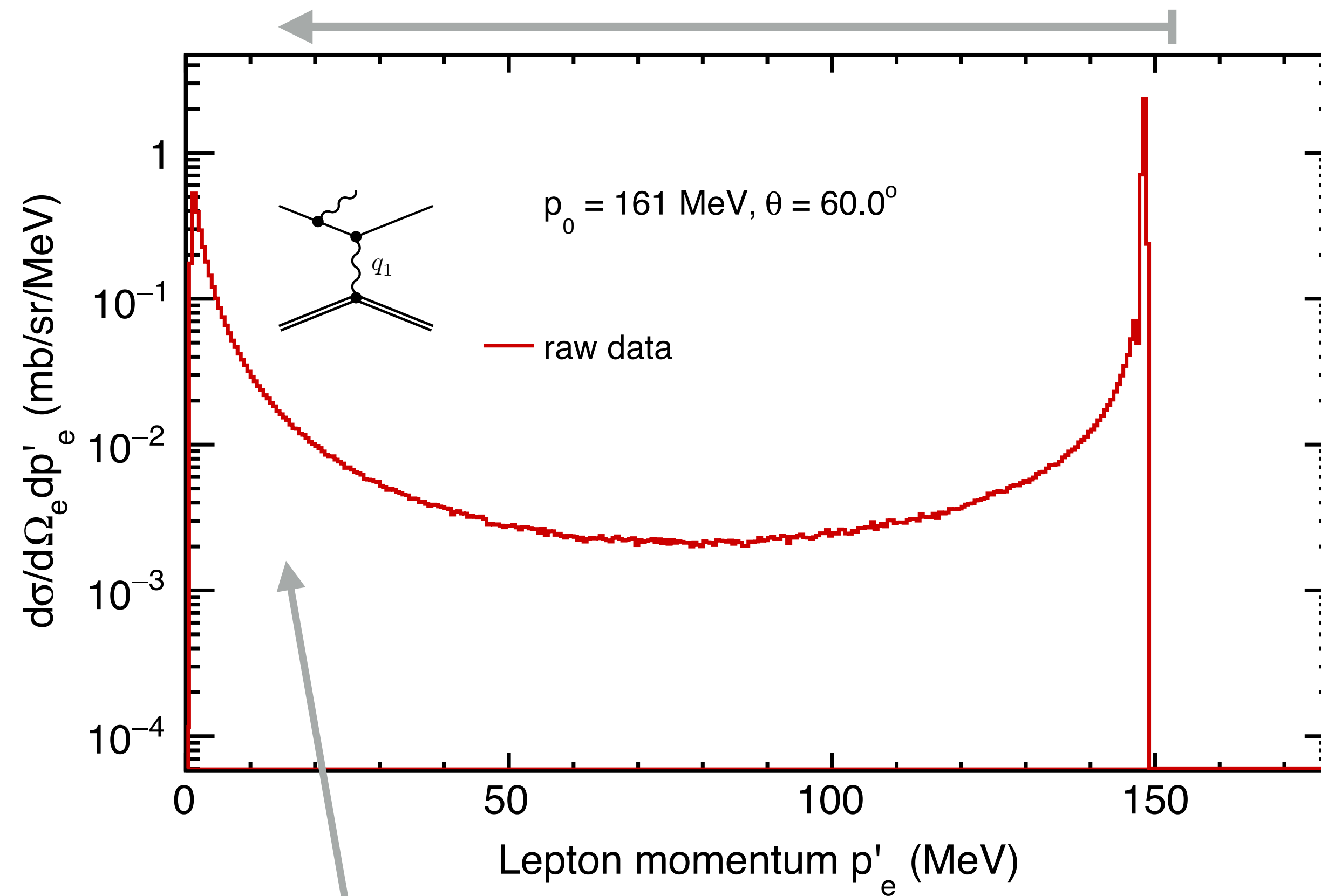
ESEPP event generator



Calorimeter
Veto on forward going
high-momentum photons

$ep \rightarrow e'p\gamma$ Cross section in MUSE kinematics

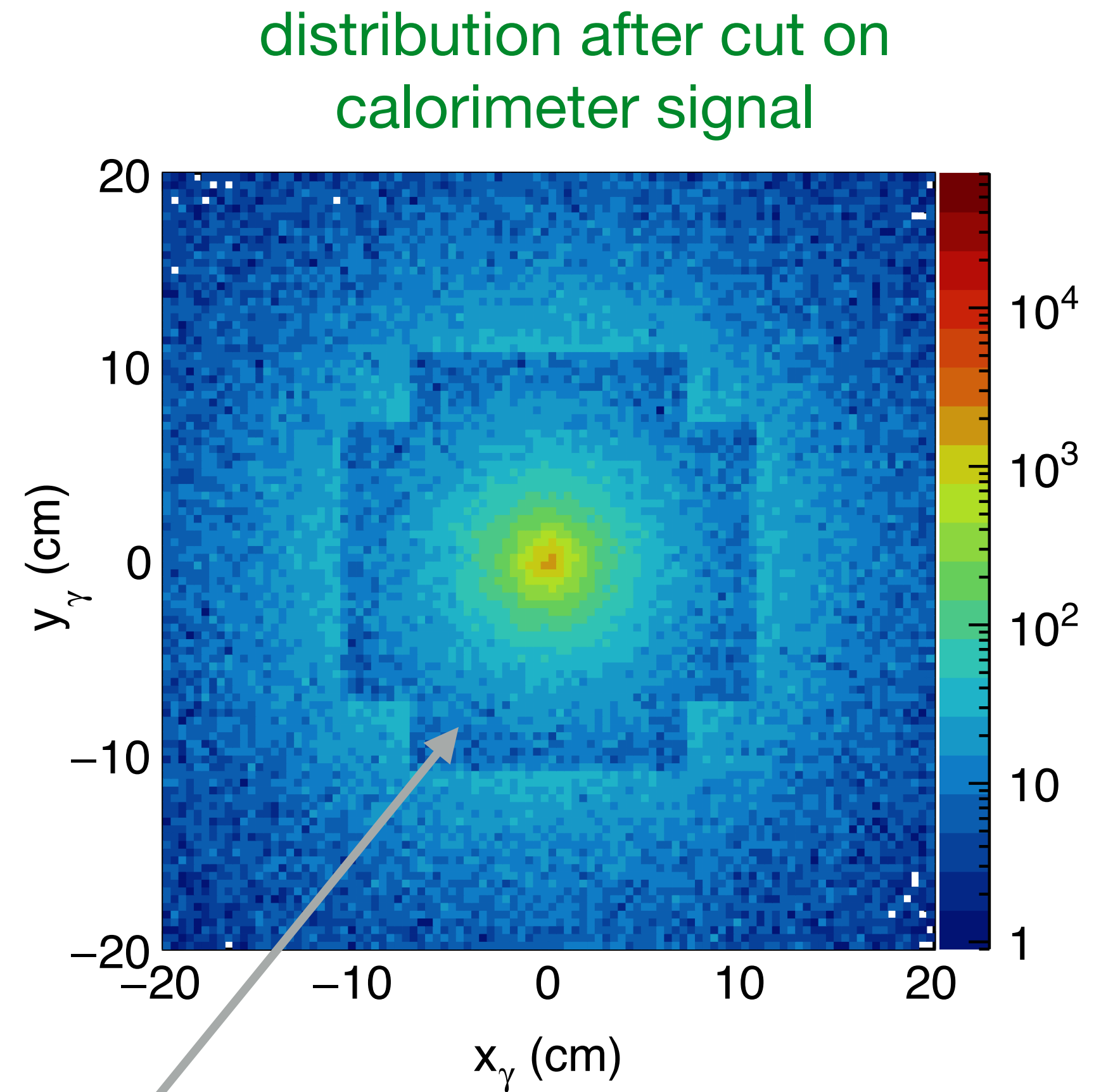
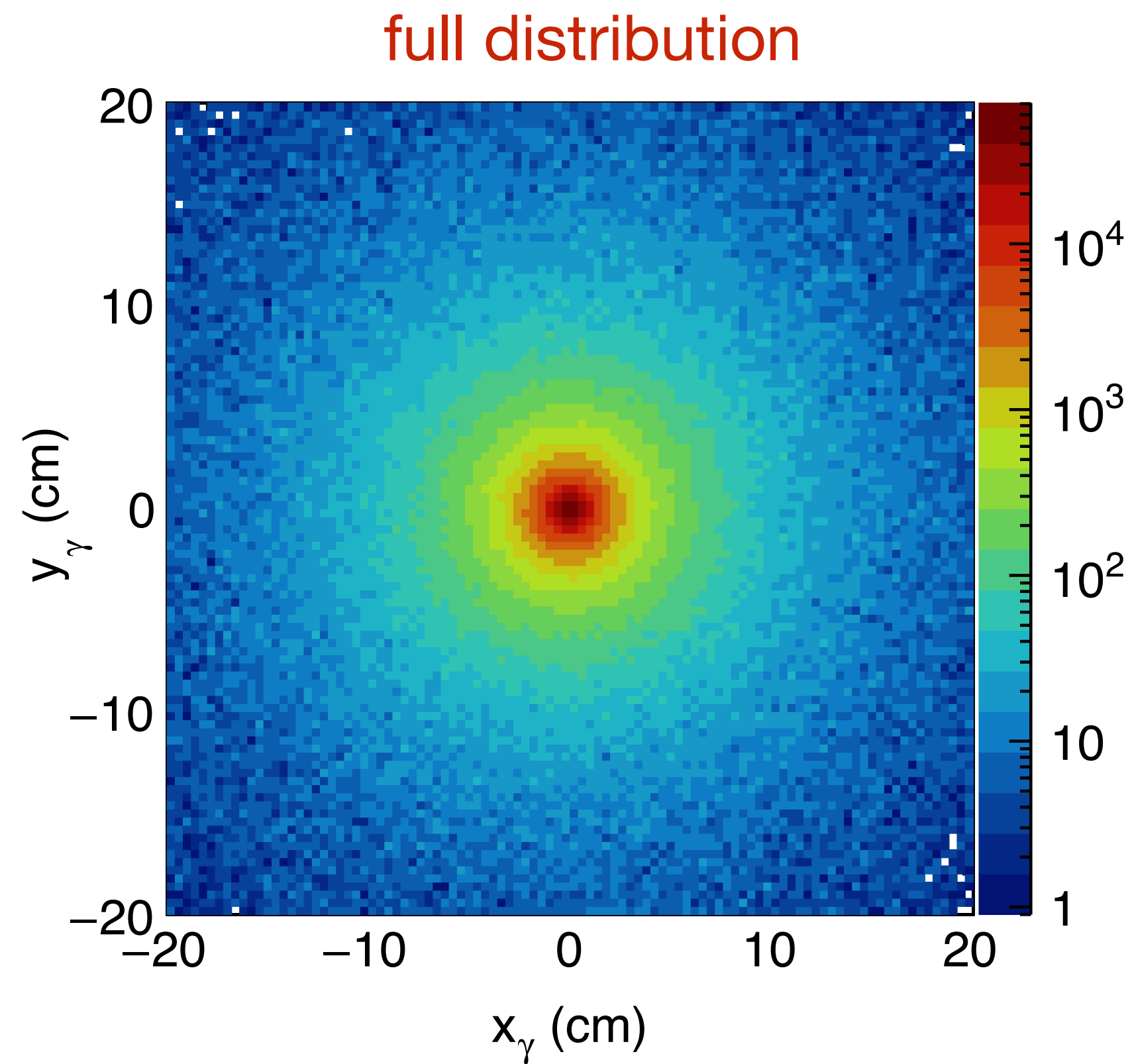
MUSE will integrate over a large momentum range



If the incident lepton loses energy due to emission of a hard photon then the probability for this lepton to be scattered by the proton increases.

Simulated downstream $ep \rightarrow e'p\gamma$ photon distribution

$p_0 = 161 \text{ MeV/c}$



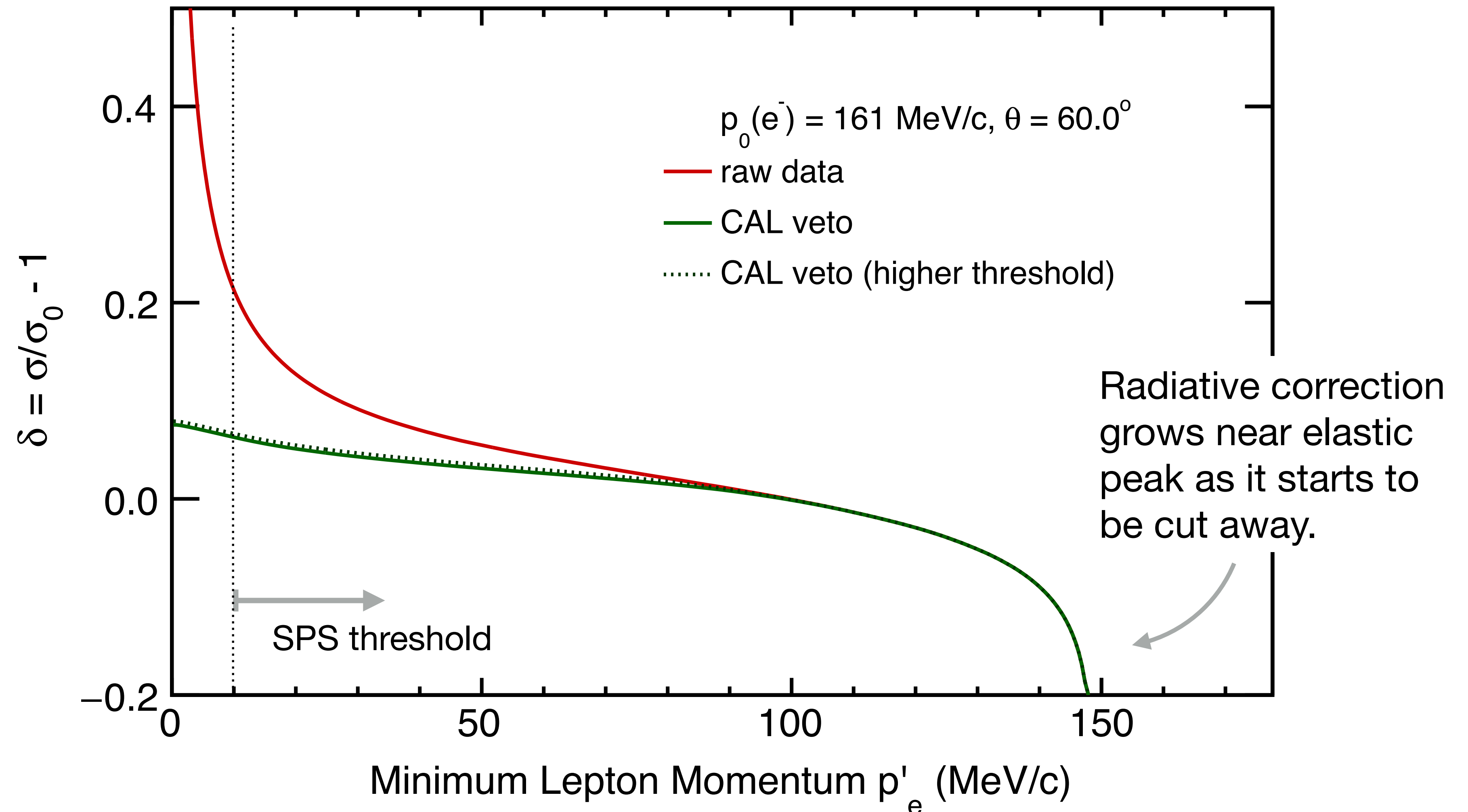
low-momentum photons,
below calorimeter threshold

Radiative corrections for electron-scattering data in MUSE kinematics

Rapidly changing
radiative corrections for
small p'_{\min} .

(> 1% change / MeV/c)

Veto on downstream
photons keep
corrections small and
reduces p'_{\min}
dependence.

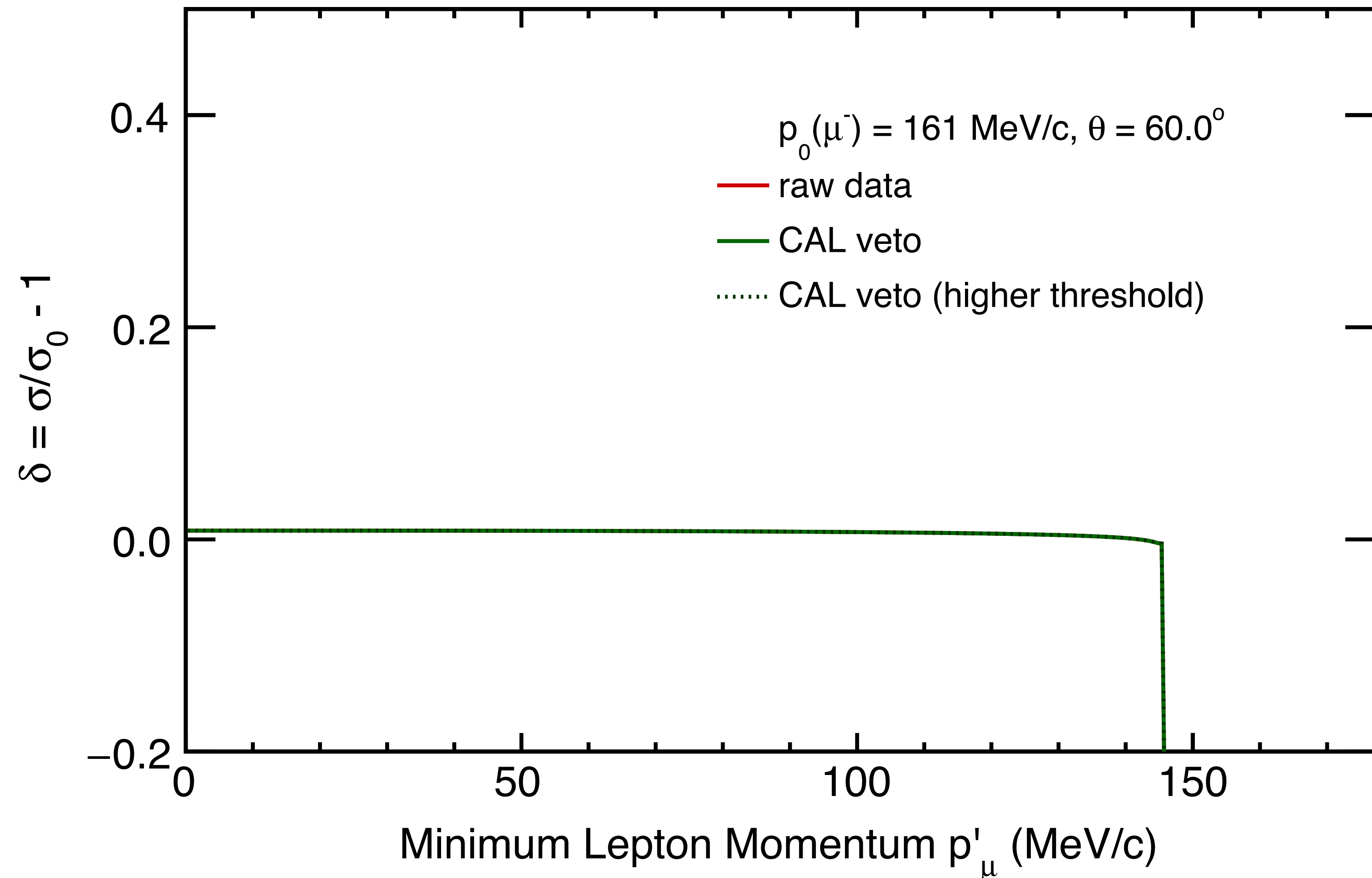


Radiative corrections for muon-scattering data in MUSE kinematics

Small radiative corrections

Corrections nearly independent of p'_{\min}

Calorimeter cut without effect on the data



Uncertainties in the radiative corrections: ep

Preliminary results

$\sigma_{\delta}(e^-)$	115 MeV			210 MeV		
	20°	60°	100°	20°	60°	100°
p'_{\min}	0.05%	0.29%	0.56%	0.03%	0.23%	0.63%
p_0	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%
θ	0.13%	0.07%	0.05%	0.10%	0.14%	0.01%
E_{γ}	0.55%	0.57%	0.58%	0.35%	0.40%	0.38%
Total	0.57%	0.65%	0.81%	0.37%	0.48%	0.74%

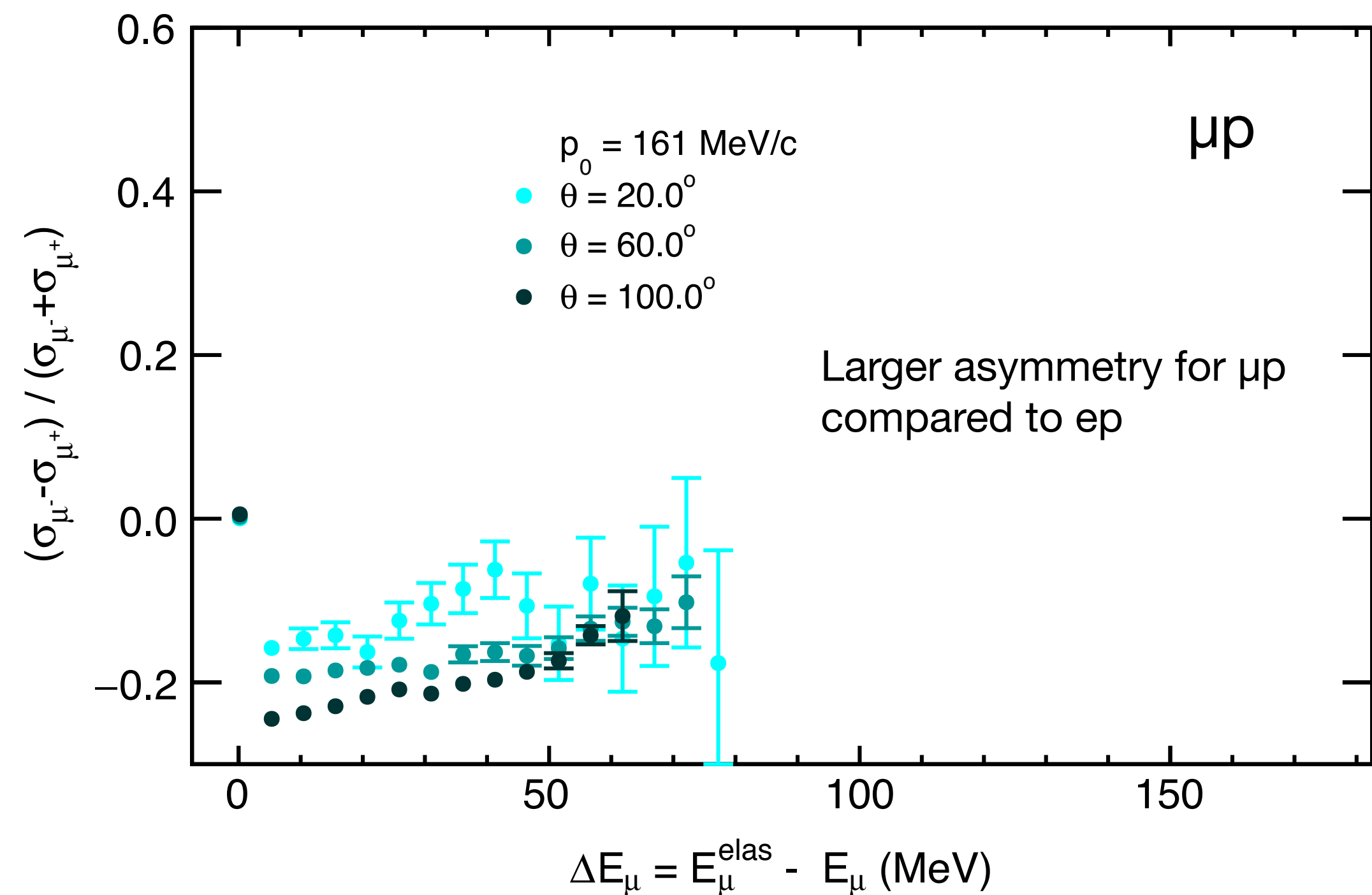
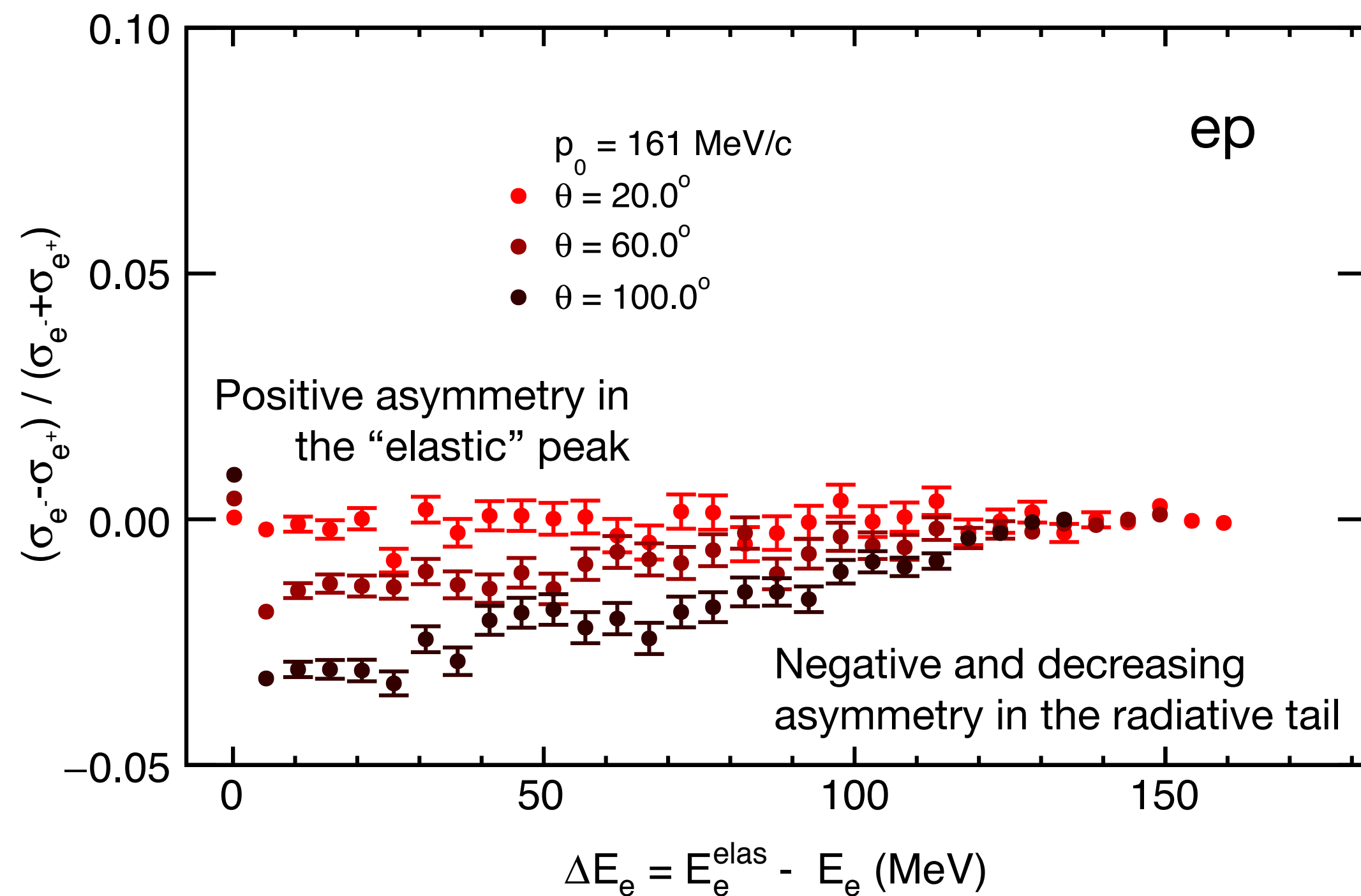
Uncertainties in the radiative corrections: μp

Preliminary results

$\sigma_{\delta}(\mu^-)$	115 MeV			210 MeV		
	20°	60°	100°	20°	60°	100°
p'_{\min}	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
p_0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
θ	0.06%	0.02%	0.04%	0.04%	0.03%	0.04%
E_{γ}	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Total	0.06%	0.02%	0.04%	0.04%	0.03%	0.04%

Cross-section asymmetries

The interference-TPE term and the interference-bremsstrahlung terms change sign depending on the sign of the lepton's charge.



More work needs to be done

- Improvement of the event sampling to more efficiently cover the full angular acceptance of the MUSE setup.
- Inclusion of the event generator in a full MUSE simulation.
- Understand the theoretical uncertainties, model dependence, and possible improvements in the calculations deep in the radiative tail.
- Separating the overall vs. the point-to-point radiative correction.
- Perform more dedicated calorimeter calibrations.

Summary

- **MUSE is a high-precision experiment** to measure the proton charge radius, study possible 2γ mechanisms, and have a direct μ/e comparison of the elastic cross-sections.
- **The MUSE setup** has unique implications for the determination of radiative corrections:
 - Without a magnetic spectrometer, MUSE does not measure the final-state lepton momentum precisely.
 - A dedicated downstream photon detector helps to suppress initial-state radiation effects.
- ESEPP simulations show **uncertainties in radiative corrections to be lower than 1 %**.